The Relevance of Learning Progressions for NAEP

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This document is one of four reports by the NAEP Validity Studies Panel that explore the relationship between NAEP and the Common Core State Standards (CCSS) and consider how NAEP can work synergistically with the CCSS assessments to provide the nation with useful information about educational progress. The complete volume with all four reports can be found at www.air.org/common_core_NAEP.

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Executive Summary

Learning progressions are one of the most important assessment design ideas to be introduced in the past decade. In the United States, several committees of the National Research Council (NRC) have argued for the use of learning progressions as a means to foster both deeper mastery of subject-matter content and higher level reasoning abilities. Consideration of learning progressions is especially important in the context of the new Common Core State Standards (CCSS) and Next Generation Science Standards (NGSS) that attend specifically to the sequencing of topics and skills across grades to ensure attainment of college and career expectations by the end of high school.

In this paper we address the question: Should more formally developed learning progressions be considered for the future design of the National Assessment of Educational Progress (NAEP)? After a brief overview of the research on learning progressions, we describe the idealized model whereby shared, instructionally grounded learning progressions—once developed—could be used to link classroom-level assessments with large-scale assessments such as NAEP. At the same time, we also consider potential problems. In particular, learning progressions—which require agreed-upon instructional sequences—could be problematic in the context of a national assessment program intended to be curriculum neutral (i.e., not favoring one state's or district's curriculum over another). Finally, we use a sample of NAEP and Balanced Assessment in Mathematics (BAM; Mathematics Assessment Resource Service, 2002, 2003) items to explore the possibility of constructing "quasi learning progressions" that could be used to illuminate the substantive meaning of the NAEP achievement results.

Can Formal Learning Progressions Be Incorporated in NAEP?

Multiple research traditions have contributed to our current understanding of learning progressions. What all of these approaches have in common is the shared understanding that learning progressions are an advancement beyond traditional curricular scope and sequence schema because they are based on research investigating and documenting how learning typically unfolds in a particular area of study. They also have either been empirically tested and revised or designed with this intent. *Thus, empirical verification and a recursive process of development are defining characteristics of learning progressions.* Importantly, these also are the features of learning progressions that ensure the close connections between assessment and instruction. Furthermore, it is because of these built-in and validated instructional supports that learning progressions hold such promise for the deepening of student learning.

The most significant impediment to implementing learning progressions for any large-scale assessment program is the fledgling state of research on learning progressions. Detailed, carefully wrought, and recursively tested progressions are rare, although the few that do exist demonstrate what is possible. A second impediment, in the case of NAEP, is the close linkage required for learning progressions between assessment tasks and instructional activities. The instructional grounding of learning progressions is a defining characteristic and core strength, but it also is a constraint if NAEP as a national assessment is required to be curriculum neutral. NAEP is intended to be an independent monitor of educational achievement in the United States over time and is used to report trends for states and important groups within the population. To enable fair comparisons, the national assessment should not favor one particular curriculum over another.[3]

If curriculum-linked learning progressions cannot be the primary or central building blocks for NAEP, the assessment must nonetheless be designed in such a way as to monitor the success of deeper curricular reforms where they occur. To continue to be an independent monitor and even a check on other assessments, NAEP must have a strategic vision that attends to both breadth and depth in representing subject-matter expertise.

In a recent white paper on the future of NAEP (National Center for Education Statistics, 2012), an expert panel recommended that NAEP domain specifications be broadened so as to enable linkages with multiple other assessments, as well as to assess advanced skills that may not be well distributed across the population. *Under such a design, the NAEP framework and reporting domain need not be the same as this comprehensive item pool, which might be thought of as a "super-assessment" domain or blueprint.* By beginning with special studies, as have been used in the past, to determine whether more advanced performance can be documented in those settings where reform curricula have been successfully implemented, assessment tasks tied to learning progressions in mathematics, science, or literacy could be embedded within the NAEP super-assessment framework. Both performance outcomes and the psychometric functioning of the assessment tasks could be compared for students with and without instructional opportunities tied directly to learning progressions curricula.

An Illustration of Quasi Learning Progressions for NAEP

The demand for curricular neutrality appears to render the use of learning progressions infeasible as a central means for developing NAEP, given the appeal of learning progressions as a way to illuminate the substantive meaning of achievement results. However, we considered the possibility of constructing "quasi learning progressions" to use as a NAEP reporting device.

Using both NAEP and BAM items, we constructed four hypothetical learning progressions representing subtopics in two of NAEP's content areas: Data Analysis and Probability, and Algebra. As a whole, BAM items are designed to tap higher levels of reasoning and application; therefore, they might be more like the kinds of assessment tasks developed to assess the CCSS.[4]

In constructing the quasi learning progressions, a critical conceptual decision was to order items by the typical instructional sequencing of topics, *not* by cognitive

[3] Many believe that adoption of the new CCSS now ensures much greater agreement among states as to how students move through topics, and thus creates the needed shared curriculum. However, a large gap remains between the general character of CCSS sequences and the specificity of actual learning progressions, which are much more dependent on specific curricular decisions.

[4] The inclusion of BAM items was possible because of an earlier study (Stancavage et al., 2009) in which NAEP and BAM items were scaled together.

complexity or perceived difficulty. The ordering process was conducted by coauthor Daro, using his knowledge of mathematics and research on mathematics learning, and reviewed by the other authors to confirm that items within each level were similar to each other in terms of the instructional topic addressed, and distinguishable from the next higher and next lower levels. We then plotted the relationship between *judged* levels of increasing proficiency on the intended construct and *empirical* evidence of item ordering for each of the four progressions, and evaluated the level of correlation between the two measures. Correlations were moderate, ranging from 0.41 to 0.60.

Based on this exercise, we conclude that such an approach is infeasible and likely to be misleading until there is more widespread implementation of the new standards and thereby greater congruence between hoped-for and empirical ordering of items. Although we can see ways to improve the meaningfulness of quasi learning progressions by eliminating misfitting items, in most cases these are not items that one would want to remove lightly. To anchor the scale with only the well-behaved items essentially moves more challenging items to a later place on the progression. These kinds of decisions can only be made after doing the kind of work that is required for the development of learning progressions (i.e., logical and expert-developed sequences must be tested in instructional contexts where students have had the opportunity to learn with the support of curricula specifically developed in conjunction with the intended progression).

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Introduction

Learning progressions are one of the most important assessment design ideas to be introduced in the past decade. The importance of their use in other countries, such as Australia and the Netherlands, reflects their fundamental characteristic, which is a much closer linkage between assessment and instruction than is true for typical large-scale assessment programs. In the United States, several committees of the National Research Council (NRC) have argued for the use of learning progressions as a means to foster both deeper mastery of subject-matter content and higher level reasoning abilities. Consideration of learning progressions is especially important in the context of the new Common Core State Standards (CCSS) and Next Generation Science Standards (NGSS) that attend specifically to the sequencing of topics and skills across grades to ensure attainment of college and career expectations by the end of high school.

Given the centrality of the CCSS and NGSS for current educational reforms, and the emphasis in these documents on the sequential deepening of content mastery and skill development over time, the question arises: Should more formally developed learning progressions be considered for the future design of the National Assessment of Educational Progress (NAEP)? In this paper, we provide a brief overview of the research on learning progressions and explain the combination of expert knowledge and empirical fieldwork needed to develop and test instructionally grounded learning progressions. We describe the idealized model whereby shared, instructionally grounded learning progressions—once developed—could be used to link classroom-level assessments with large-scale assessments such as NAEP. At the same time, we also consider potential problems. In particular, learning progressions—which require agreed-upon instructional sequences—could be problematic in the context of a national assessment program intended to be curriculum neutral (i.e., not favoring one state's or district's curriculum over another).

Due to the potential appeal of learning progressions as a way to illuminate the substantive meaning of achievement results, in this report we consider the possibility of constructing "quasi learning progressions" as a reporting device. We call them quasi progressions because they are developed after the fact, rather than being jointly constructed and field tested as a continuum of instructional and assessment tasks. Using data from a previous NAEP Validity Studies Panel investigation and an approach similar to the anchoring methodology used earlier in NAEP's history, we construct three quasi learning progressions for eighth-grade mathematics. This exercise illustrates the potential benefits of using sequenced exemplar items to give meaning to the numerical score scale. At the same time, misfitting items illustrate the difficulty of meeting both the logical and empirical requirements of learning progressions in multidimensional assessment domains.

Definition

Learning progressions are known by various terms: progress maps, progress variables, developmental continua, progressions of developing competence, profile strands, learning trajectories, and learning lines. According to Masters and Forster (1996, p. 4), "A progress map describes the knowledge, skills and understandings of a learning area in the sequence in which they typically develop and provides examples of the kinds of performances and student work typically observed at particular levels of attainment." Similarly, in *Taking Science to School: Learning and Teaching Science in Grades K–8*, learning progressions were defined as "descriptions of the successively more sophisticated ways of thinking about a topic that can follow one another as children learn about and investigate a topic over a broad span of time" (Duschl, Schweingruber, & Shouse, 2007). Although order is an implied characteristic of learning progressions, making it possible to quantify increases in proficiency, learning progressions are distinguished from other score scales by their attention to substantive markers of increasing proficiency. They are "criterion-referenced," in Glaser's (1963) original sense of the term, meaning that they are grounded in actual criterion performance and illustrate explicitly how performance has to improve to move higher on the score scale.

In part because of their sudden popularity and also because of their emergence in very different research literatures, the idea of learning progressions cannot be reduced to a single agreed-upon and precise definition. Early work in Australia using the term "progress maps" was informed by Rasch model scaling and therefore attended more to psychometric requirements (Masters, Adams, & Wilson, 1990; West Australian Ministry of Education, 1991). Other early work, also in Australia and the United States, focused on emergent literacy and was similar to parallel work in the United States examining early childhood mathematics learning. These latter efforts focused on instructional tasks that could be ordered on a continuum that also served assessment purposes (Baroody, 1984; Fuson, 1992). Some learning progressions are quite broad and general, depicting the mastery of a content domain over several grade levels. Other learning progressions are very detailed and focus on increasing mastery within a single unit of instruction. In the earliest grades, progressions may be affected by biological development, although the rate at which children proceed can clearly be influenced by instructional supports. Most learning progressions do not, however, imply some underlying latent trait. Rather, they reflect curricular and instructional choices within which may lie some "natural" orderings of difficulty. For example, multiplication may be easier than subtraction, depending on how they are taught, but two-digit subtraction will nearly always be easier than three-digit subtraction.

Why the Appeal? Learning Progressions in the Context of the Common Core

Unlike standards documents from the early 1990s that emphasized what students should "know and be able to do" at a given grade level, the CCSS are oriented toward cumulative growth in knowledge and skills across grade levels. The English language arts grade-level standards, for example, "define end-of-year expectations and a cumulative progression" leading to college and career readiness (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010a, p. 4). The specific reading standards establish "a grade-by-grade 'staircase' of increasing text complexity that rises from beginning reading to the college and career readiness level" (p. 8). Similarly, authors of the mathematics standards attended both to the hierarchical logic of disciplinary structures and to research on "how students' mathematical knowledge, skill, and understanding develop over time" (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010b, p. 4), with the intention of empirically verifying these sequences even more rigorously in the future.

Some of the popular rhetoric surrounding the CCSS makes it appear as if the sequential nature of the standards arose primarily from an exercise in backwards planning intended to ensure arrival at the endpoint of college and career readiness. Unfortunately, using "college and career readiness" as a short-hand summary for learning goals sometimes obscures the important underlying reform principle that links sequencing of learning goals with the need for greater rigor and depth of understanding. Most policymakers today are familiar with findings from more than a decade ago that attributed the poor performance of U.S. students on international comparisons to our "mile-wide and inch-deep curricula" (Schmidt, McKnight, & Raizen, 1997). In subsequent investigations, Schmidt and colleagues identified the features of "curriculum coherence" that distinguished the curricula of top-performing nations from the unfocused and repetitive curricula fostered by U.S. state and district standards documents. Surprisingly, for those who assume that academic excellence requires covering *more* topics, curriculum documents from high-performing countries included fewer topics per grade than is typical of U.S. standards because in high-performing countries topics were introduced, studied in greater depth, and then intentionally removed from the curriculum. In contrast, topics "linger" in U.S. curricula once they are introduced.

Fewer topics in the A+-rated countries naturally implied more focus. More importantly, however, the sequencing of topics in high-performing countries also appeared to be more carefully orchestrated to build on concepts from one grade to the next. Schmidt, Wang, and McNight (2005) concluded that standards meet a criterion of coherence "if they specify topics, including the depth at which the topic is to be studied as well as the sequencing of the topics, both within each grade and across the grades, in a way that is consistent with the structure of the underlying discipline" (p. 554). A basic goal of the CCSS is not only to design the standards to reflect the structure of the discipline or skill dimension, but also to make this structure visible to students as part of their understanding and mastery of the subject matter.

A word of caution is required, however, before assuming that the CCSS meet a technical definition of formal learning progressions. The same must be said of the NGSS despite their focus on core ideas that are "teachable and learnable over multiple grades at increasing levels of depth and sophistication." As we explain in the next section, elaborating within the broader standards frameworks to establish formal learning progressions will require a much more detailed codevelopment of instructional and assessment materials based on both expert judgment and empirical verification. Authors of the CCSS are aware that local variability and limitations in the research base make it impossible to say with certainty that topic A should always come before topic B. In describing the CCSS in mathematics, they note the following:

> …*grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today.* (Common Core State Standards Initiative, 2012)

Thus, it might be useful to think of the grade-to-grade continua underlying the CCSS and NGSS as "learning sequences" and reserve the term learning progressions for more carefully developed progressions that meet the technical definition. Or, at a minimum, given the popular and pervasive use of "learning progressions" talk, it should be acknowledged that Common Core progressions are hypothetical and preliminary and are expected to be refined by further research and development.

Instructional Benefits and Requirements for the Development of Learning Progressions

The sudden policy interest in learning progressions as a reform strategy has led to some confusion about terminology and, more fundamentally, about the defining characteristics of learning progressions and what they can promise to do. This is due largely to the rapid merging and comingling of multiple research traditions. For example, mathematics education and science education have distinct research literatures, respectively, on learning trajectories and learning progressions. Some approaches to learning progressions have a decidedly measurement or assessment focus, meaning that the goal of research projects in this tradition is to produce a specific measurement instrument. Other approaches come from contemporary improvements in learning research—focusing on children's thinking and the need to design instructional tasks that directly build on students' intuitive understandings and prior experiences, but without attempting to score or quantify the level of student attainment. Assessment may be nearly invisible in the latter case. Some progressions are quite general and cover broad age spans as is intended for the CCSS. Examples provided by Masters and Forster (1996) are from national curricula for England and Wales, Australia, Hong Kong, and Canadian provinces. Other progressions, such as the "Sinking and Floating" example developed at the Stanford Education Assessment Laboratory (Ayala et al., 2008), mark progress over a single unit of study.

What all of these approaches have in common is the shared understanding that learning progressions are an advancement beyond traditional curricular scope and sequence schema because they are based on research investigating and documenting how learning typically unfolds in a particular area of study. They also have either been empirically tested and revised or designed with this intent. *Thus, empirical verification and a recursive process of development are defining characteristics of learning progressions.* Importantly, these are also the features of learning progressions that ensure the close connections between assessment and instruction. Furthermore, it is because of these built-in and validated instructional supports that learning progressions hold such promise for the deepening of student learning.

In a recent report summarizing research on learning progressions in science, Corcoran, Mosher, and Rogat (2009) identified five essential components of learning progressions as shown in Table 1.

Table 1. Essential Components of Learning Progressions

1.	Learning targets or clear end points that are defined by societal aspirations and analysis of the central concepts and themes in a discipline;
2.	Progress variables that identify the critical dimensions of understanding and skill that are being developed over time;
3.	Levels of achievement or stages of progress that define significant intermediate steps in conceptual/skill development that most children might be expected to pass through on the path to attaining the desired proficiency;
4.	Learning performances which are the operational definitions of what children's understanding and skills would look like at each of these stages of progress, and which provide the specifications for the development of assessments and activities which would locate where students are in their progress; and,
5.	Assessments that measure student understanding of the key concepts or practices and can track their developmental progress over time.

Source: Corcoran et al., 2009, p. 15.

This distillation makes a useful distinction between the encompassing term, learning progressions, and the more detailed specification of skills required for "progress variables" as noted in step 2. In calling out these steps, the authors drew from the grand conceptual steps (steps 1 and 3) laid out in *Taking Science to School*, and the more detailed steps followed by Smith, Wiser, Anderson, and Krajcik (2006), to create progress variables, learning performances, and assessments of key concepts and practices in their construction of a learning progression for matter and the atomic-molecular theory. To be complete, we note that the conceptual steps described in *Taking Science to School* begin with a prior step that "anchors" learning progressions at one end "by what is known about the concepts and reasoning of students entering school" (Corcoran et al., 2009, p. 219).

As part of their synthesis project, Corcoran et al. (2009) and their panel of experts identified further the following possible benefits of learning progressions, which again emphasized the coconstruction of instructional materials and assessment tasks.

- They should provide a more understandable basis for setting standards, with tighter and clearer ties to the instruction that would enable students to meet them;
- They would provide reference points for assessments that report in terms of levels of progress (and problems) and signal to teachers where their students are, when they need intervention, and what kinds of intervention or ongoing support they need;
- They would inform the design of curricula that are efficiently aligned with what students need to progress;
- They would provide a more stable conception of the goals and required sequences of instruction as a basis for designing both pre- and in-service teacher education.
- The empirical evidence on the relationship between students' instructional experiences and the resources made available to them, and the rates at which they move along the progressions, gathered during their development and

> ongoing validation, can form the basis for a fairer set of expectations for what students and teachers should be able to accomplish, and thus a fairer basis for designing accountability systems and requirements. (pp. 9–10)

An example from mathematics serves to highlight the grounding of learning progressions in children's thinking and their subsequent linking to instructional interventions. Drawing on their own work for more than a decade and that of others, Clements and Sarama (2009) described early childhood mathematics progressions (trajectories) for counting, early arithmetic, spatial thinking, geometric shapes, and geometric measurement (e.g., length, area). Their work is instructive because it illustrates both the research process needed to develop learning progressions and the subsequent use of progressions to support and thereby accelerate and deepen student learning.

The learning progression for counting from Clements and Sarama (2009) is presented in Table 2. They note that this progression comprises three subtrajectories: verbal counting (knowing the number names), object counting, and counting strategies. These three subtrajectories build from one to the next but also become increasingly interrelated. "To count a set of objects, children must not only know verbal counting but must also learn (a) to coordinate verbal counting with objects by pointing to or moving the objects and (b) that the last counting word names the cardinality of ('how many objects in') the set" (p. 21). To establish the steps or levels in the progression, the researchers synthesized clinical interview and observational findings from dozens of prior studies. They developed descriptive labels and recognizable counting behavioral markers for each step. Then, importantly, Clements and Sarama developed instructional tasks for each level that would foster the kind of thinking required at that level. For example, as most parents know, touching each object while counting helps move "reciters" to the next level, "corresponders." To move from counting to understanding the "how many" question (the cardinality principle), children are first asked, "how many do I have?" after one object is added or removed. Next, they are asked "how many?" with surprise additions or subtractions of two or three.

Table 2. Learning Trajectory for Counting

Pre-Counter *Verbal* No verbal counting. Names some number words with no sequence. **Chanter** *Verbal* Chants "sing-song" or sometime indistinguishable number words.
Reciter *Verbal* Verbally counts with separate words, not necessarily in the correct order above "five."
Reciter (10) *Verbal* Verbally counts to ten, with *some* correspondence with objects, but may either continue an overly rigid correspondence or exhibit performance errors (e.g., skipping, double-counting). **Corresponder** Keeps one-to-one correspondence between counting words and objects (one word for each object), at least for small groups of objects laid in a line. May answer a "how many?" question by re-counting the objects, or violate 1-1 or word order to make the last number word be the desired or predicted word.
Counter (Small Numbers) Accurately counts objects in a line to 5 and answers the "how many" question with the last number counted. When objects are visible, and especially with small numbers, begins to understand cardinality. **Counter (10)** Counts arrangements of objects to 10. May be able to write numerals to represent 1–10. Accurately counts a line of 9 blocks and says there are nine. Verbal counting to 20 is developing.
Producer (Small Numbers) Counts out objects to 5. Recognizes that counting is relevant to situations in which a certain number must be placed. Produces a group of 4 objects.
Counter and Producer (10+) Counts and counts out objects accurately to 10, then beyond (to about 30). Has explicit understanding of cardinality (how numbers tell how many). Keeps track of objects that have and have not been counted, even in different arrangements. Writes or draws to represent 1 to 10 (then 20, then 30).
Counter Backward from 10 *Verbal and Object*
Counter from N (N + 1, N – 1) *Verbal and Object* Counts verbally and with objects from numbers other than 1 (but does not yet keep track of the *number* of counts).
Skip Counter by 10s to 100 *Verbal and Object* Skip counts by tens up to 100 or beyond with understanding; e.g., "sees" groups of 10 within a quantity and counts those groups by 10 (this relates to multiplication and algebraic thinking).
Counter to 100 *Verbal* Counts to 100. Makes decade transitions (e.g., from 29 to 30) starting at any number.
Counter On Using Patterns *Strategy* Keeps track of a few counting acts, but only by using numerical patterns.
Skip Counter *Verbal and Object* Counts by fives and twos with understanding.
Counter of Imagined Items *Strategy* Counts mental images of hidden objects.
Counter On Keeping Track *Strategy* Keeps track of counting acts numerically, first with objects, then by "counting counts." Counts up 1 to 4 *more* from a given number.
Counter of Quantitative Units/Place Value Understands the base-ten numeration system and place-value concepts, including ideas of counting in units and multiples of hundreds, tens, and ones. When counting groups of 10, can decompose into 10 ones if that is useful.
Counter to 200 *Verbal and Object* Counts accurately to 200 and beyond, recognizing the patterns of ones, tens, and hundreds.
Number Conserver Consistently conserves number (i.e., believes number has been unchanged) even in face of perceptual distractions such as spreading out objects of a collection.
Counter Forward and Back *Strategy* Counts "counting words" (single sequence or skip counts) in either direction. Recognizes that decades sequence mirrors single-digit sequence.

Source: Clements & Sarama, 2009, pp. 30-41.

Clements and Sarama (2007a) used this extensive program of research to develop the Building Blocks curriculum and computer software to support learning in both early numeracy and geometry. The impact on student learning of carefully designed interventions tailored to specific levels of learning progressions was documented in a comparative study conducted in preschool programs serving low-income families (Clements & Sarama, 2007b). Within state-funded preschool and Head Start school sites, classrooms were assigned to treatment or control groups. Control classrooms continued to receive the existing preschool curriculum. Participants were assessed at the beginning and end of the school year using individual interview protocols designed to cover the same topics as the curriculum but without mirroring the instructional activities. The statistical and practical significance of the effects was dramatic. For the Number and Geometry outcome measures, the effect-size differences between the treatment and control groups at the time of the post assessment were .85 and 1.47, respectively. Similar effects were also obtained for differential gains from pre- to post-assessment for the treatment group compared with the control group. The fact that instructional supports targeted to each level of the progressions were so effective provides additional evidence as to the validity of the progressions.

Clements and Sarama (2009) describe their progressions as *developmental progressions,* meaning that they represent natural sequences that are affected by biology. They use the example of infants and children first learning to crawl, then walk, then run, skip, and jump. Although biological readiness may also affect the *order* of skill development in mathematics and other early learning, Clements and Sarama (2009) emphasize that development may be fast or slow depending on learning opportunities. Many decades ago psychologists believed that development proceeded at a fixed pace and could not be hurried. On the contrary, contemporary learning research has demonstrated that learning affects and interacts with development—hence the interest in instructional moves specifically targeted to developmental stages. Virtually all researchers studying learning progressions recognize that development is strongly affected by learning opportunities and specific instructional contexts. As noted by Masters and Forster (1996), a learning progression is "NOT a description of 'natural' sequences of development only. A progress map is the result both of 'natural' sequences of student development and common conventions for the content and delivery of curricula, and may be elucidated by systematic research into student learning" (p. 11).

In addition to guiding instructional interventions, other potential benefits of learning progressions are more directly applicable to large-scale assessment applications. However, these benefits also derive from the connectedness of learning progressions to particular instructional practices. The NRC report, *Knowing What Students Know* (Pellegrino, Chudowsky, & Glaser, 2001), outlined key requirements for reforming assessment systems if they are to capitalize on recent findings from cognitive science research and measurement theory. Of their three requirements for assessment systems—comprehensiveness, coherence, and continuity—the latter two can best be met by the use of learning progressions. *Comprehensiveness* refers to the completeness with which various learning goals are represented by the assessment system. *Coherence* addresses the relationship among assessments at different levels of the system. In the past, large-scale assessments have been misaligned with classroom tasks and learning goals or, when they were made coherent, it was by creating classroom work and assessments that imitated external tests. If classroom formative assessments and large-scale assessments were designed around shared learning progressions instead, the resulting system would be conceptually coherent even if classroom materials would need to be developed at a much finer grain size. Last, *Knowing What Students Know* (Pellegrino, et al., 2001) recommended that ideal assessment systems be designed to be *continuous* as follows:

> *Assessments should measure student progress over time, akin more to a videotape record than to the snapshots provided by the current system of on-demand tests. To provide such pictures of progress, multiple sets of observations over time must be linked conceptually so that change can be observed and interpreted. Models of student progression in learning [emphasis added] should underlie the assessment system, and tests should be designed to provide information that maps back to the progression. With such a system, we would move from "one-shot" testing situations and cross-sectional approaches for defining student performance toward an approach that focused on the processes of learning and an individual's progress through that process. (pp. 256–257)*

Imagine a coherent and continuous system whereby classroom instructional activities and formative assessment tasks are developed, in tandem, as part of a learning progression. Then when it comes time to build the large-scale assessment, representative tasks are developed to measure progress along that same learning progression. Forster and Masters (2004) described just such a system developed by the Australian Council for Educational Research (ACER). They confess that they did not set out initially to build both classroom-level and linked national assessments, but having done so, they make a strong case for the resulting synergies and coherence. Their national survey assessment was built subsequent to the development of classroom-level curriculum and assessment materials but was closely tied to them, using the same underlying progressions.

ACER first created a Developmental Assessment Resource for Teachers (DART) "to assist teachers in assessing students' knowledge, skills, and understandings in English (language arts) at the elementary (Australian 'primary') level" (p. 52). Although the emphasis was on helping teachers to assess students' classroom work by providing assessment tasks, scoring guides, and samples of student work, the nature of the project also helped teachers develop a deep and shared understanding of the new national English curriculum framework that had been released that same year. Assessment materials were designed around common themes, videotapes were provided to set the theme, and teachers were encouraged to develop their own materials consistent with the theme. Later, the National School English Literacy Survey (NSELS) was developed based on the DART model and was able to use the same mix of classroom-based, teacher-scored authentic literacy tasks. In addition, because of a shared curriculum, the national survey could use tasks that called on the same themes as the classroom-level assessments. For example, a Year 3 poem on the NSELS about mosquitoes related to a film that children had watched as part of the DART myths and legends theme. To ensure reliability and comparability, external assessors joined teachers in scoring, but the national survey tasks were still highly congruent with typical classroom practices. According to Forster and Masters (2004), progress maps for each of the skill areas (reading, writing, spelling, and speaking) provided the "conceptual backbone" that made possible this kind of coherence between their classroom-level and accountability assessments.

Challenges to Implementing Learning Progressions With NAEP

The most significant impediment to implementing learning progressions for any large-scale assessment program is the fledgling state of research on learning progressions. Clements and Sarama's (2009) detailed, carefully wrought, and recursively tested early mathematics progressions are rare. They are an existence proof demonstrating what is possible, but similarly created progressions do not exist across grades and subject matters. Several progressions have been constructed in the sciences for matter and atomic molecular theory (Smith et al., 2006), evolution (Catley, Lehrer, & Reiser, 2004), complex reasoning about biodiversity (Songer, Kelcey, & Gotwals, 2009), force and motion (Alonzo & Steedle, 2009), genetics (Duncan, Rogat, & Yarden, 2009), and carbon cycling in socioecological systems (Mohan, Chen, & Anderson, 2009). Note that, as with all progressions, these are acknowledged to be working hypotheses or draft progressions. They are research based in that prior evidence and experience supports the reasoning that went into authoring the progressions. They have also been field tested, in many cases undergoing multiple iterations and revisions. However, although these development projects reflect the integration of big ideas and practices that are called for in the NGSS, they still have not worked out how multiple progressions of this type would be brought together in a coherent curriculum. The learning sequences embedded in the CCSS are even less well developed. They are research based in the sense that they use available research evidence about which concepts appear easier than others and, once mastered, facilitate subsequent learning. Expert judgment has been used to fill in the gaps. But the CCSS have not been empirically tested as to the rate at which progress is likely to occur and with what affordances, nor is there research knowledge yet about concurrent pursuit of these standards and the extent to which concurrence might foster (or impede) joint progress.

A second impediment, in the case of NAEP, is the close linkage required for learning progressions between assessment tasks and instructional activities. The instructional grounding of learning progressions is a defining characteristic and core strength, but it is also a constraint if NAEP as a national assessment is required to be curriculum neutral. NAEP is intended to be an independent monitor of educational achievement in the United States over time and is used to report trends for states and important groups within the population. To enable fair comparisons, the national assessment should not favor one particular curriculum over another. Therefore, it could not base its frameworks on specific curriculum-based progressions. In the past, we have argued that the national assessment should be comprehensive, reflecting the union of multiple curricular approaches (National Academy of Education Panel on the Evaluation of the NAEP Trial State Assessment, 1992), and, indeed, although not as broad as the sum of all possible state frameworks, NAEP has been found to have greater reach in terms of cognitive complexity than many state assessments (Daro, Stancavage, Ortega, DeStefano, & Linn, 2007). Now, in the context of the CCSS, continuing to envision NAEP as the union of multiple curricula could contribute to a milewide, inch-deep problem if

NAEP does not explicitly attend to the depth-over-breadth conception of advanced performance.

Many believe that adoption of the new CCSS now ensures much greater agreement among states as to how students move through topics, and thus creates the needed shared curriculum. However, a large gap remains between the general character of CCSS sequences and the specificity of actual learning progressions, which are much more dependent on specific curricular decisions. The gap between general frameworks and specific curricula is particularly great if the intent of both is to aim for deeper understanding rather than superficial coverage. The ability to ask for deeper understanding, for example, in comparing character development in two different works of fiction requires that the test maker know what novels students have read. The demands of "going deeper" are especially great if we take seriously the relatively old finding from cognitive science research that thinking skills cannot be developed independent of content. When applied specifically to the NGSS and research on learning progressions in the sciences, this means that topics must be integrated with scientific practices; there are many ways of doing this that would still be consistent with the NGSS. Citing the *Taking Science to School* definition of learning progressions, Songer et al. (2009) argue that "successively more sophisticated ways of thinking about a topic…recognizes the inherent presence and interconnection of content knowledge with inquiry reasoning" (p. 611). In their development of a learning progression for complex reasoning about biodiversity, Songer et al. (2009) paired a biodiversity continuum (from "plants and animals differ" to "taxonomic diversity and abundance") with an inquiry reasoning progression based on evidence-based explanations. Had they picked "planning and carrying out investigations" or "analyzing and interpreting data"—other scientific practices that also require complex reasoning—the assessment and curricular tasks at the higher end of the progression would have looked quite different.

NAEP's History With Related Item-Anchoring Methodologies

When item response theory (IRT) was first introduced in the field of measurement, and later adopted as a NAEP's primary analytic model, one of its most desirable features was its ability to locate examinees and items on the same score continuum—thus making it possible to offer criterion-referenced interpretations of examinee's scores. Unfortunately, as researchers quickly realized, making statements about what examinees at a given score level "can do" depended greatly on the orderliness of the items being scaled, the criterion used to locate items on the scale, and the degree of relationship between unique items and more general descriptions of competencies. When they have not been specially designed to reflect sequential mastery, items do not march up the score continuum in tidy increments. The notion of a Guttman (1950) scale, whereby examinees can be located so that they fail all of the items above them on the scale and answer perfectly all of the items below them, simply does not occur in the world of achievement testing.

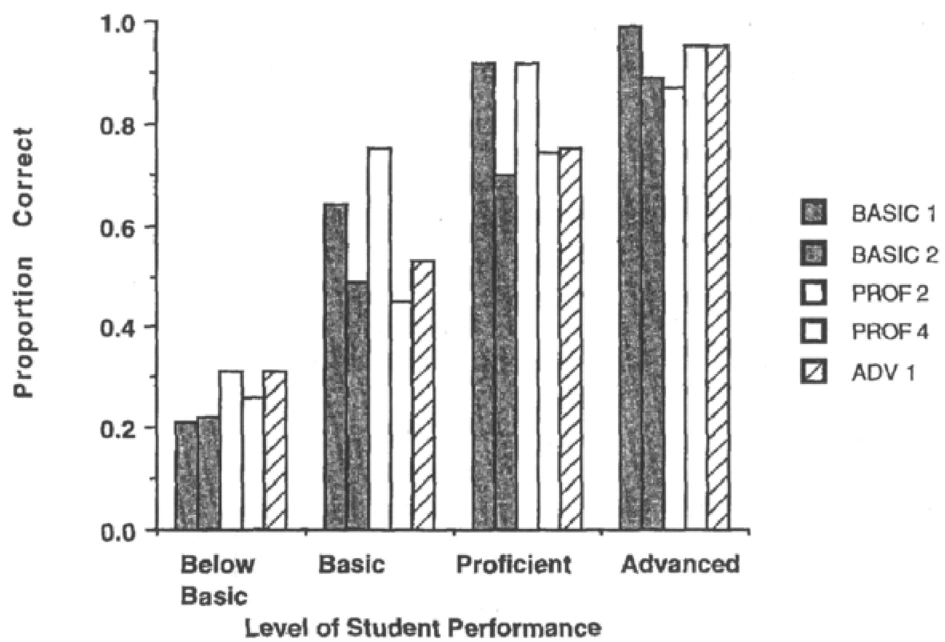
As described by Beaton and Allen (1992), item-anchoring methods were developed to identify the types of items that characterized performance at anchor points on the NAEP scale (150, 200, 250, 300, 350). The steps involved in creating anchor descriptions are as follows:

1. Form groups of examinees in close proximity to each anchor point.
2. For each item at each anchor point, calculate the proportion correct for the proximal group.
3. For each anchor point, determine which items could be answered correctly by a substantial majority of students at that level.
4. For succeeding anchor points, determine which items could be answered correctly by a substantial majority of examinees at that level but not by most of the students at the level of the next lower anchor point.
5. Given the sets of items identified at each anchor point, develop generalizations to describe the performance level characterized by these items.

In one of the earliest critiques of item-derived anchoring and criterion-referenced interpretations, Forsyth (1991) argued that in complex domains, such as NAEP mathematics and science assessments, learning could not possibly be expected to proceed uniformly for all examinees due to the different combinations of content, context, and cognitive processes. "Test developers face the enormous problems created by the interaction of an examinee's past experiences and the content of the item" (p. 5). Forsyth provided numerous examples of misinterpretations that were likely to occur because of the multidimensional nature of NAEP's composite scales. Most famously, Shanker (1990) assumed that only 6 percent of 17-year-olds could solve multistep math problems because such an item was used to anchor the 350-scale point, and only 6 percent of 17-year-olds scored above 350.

Linn (1998) further described the variations in item difficulties that could occur, not because of the level of proficiency associated with the skill or construct the item was intending to measure, but because of the particular question asked, the wording of distracters, and scoring rubrics in the case of open-ended questions. As an example, Linn noted the pattern shown in Figure 1 from Burstein et al. (1995/1996). When exemplar items were selected to illustrate the verbal descriptions of the 1992 mathematics achievement levels, the figure shows that, in some cases, a majority of students at a particular level could not answer an exemplar item selected for that level. The converse was also sometimes true, as when 77 percent of Basic-level students could answer one of the Proficient-level exemplars correctly and 79 percent of Proficient-level students could answer the Advanced-level exemplar correctly. As Linn notes, these obvious types of errors were eliminated in subsequent NAEP reports by applying statistical criteria in addition to logically matching items to verbal descriptions.

Figure 1. Proportion Correct by Achievement Level for Grade 4 Exemplar Items Selected to Illustrate Proficient and Advanced Exemplars That are Statistically Similar to Basic Exemplars



Source: Linn, 1998. Reprinted by permission of Taylor & Francis (http://www.tandfonline.com).

More recently, Schulz, Lee, and Mullen (2005) summarized difficulties with prior attempts to use individual items to make criterion-referenced descriptions of achievement and then proposed an alternative method using substantively identified testlets or domains of NAEP items that could be instructionally ordered. Using this method with eighth-grade NAEP mathematics data from 2000, they were able to show that performance on these expert- and teacher-identified domain-testlets was consistent with their expected instructional sequencing. Although these ordered domains do not have the detail of closely developed, curriculum-specific learning progressions, they do comport well with the broader grade-to-grade "progressions" envisioned for the CCSS, and therefore might well be a reasonable methodology to use with NAEP to help with scale interpretations.

We did not attempt to implement the Schulz et al. (2005) methodology for this paper because of cost constraints and because investment in such a study would make more sense sometime after the instructional sequencing based on the CCSS could reasonably be expected to be implemented. Nonetheless, for future reference, it may be useful here to elaborate on key features of the Schulz et al. methodology as distinct from item-anchoring methods.

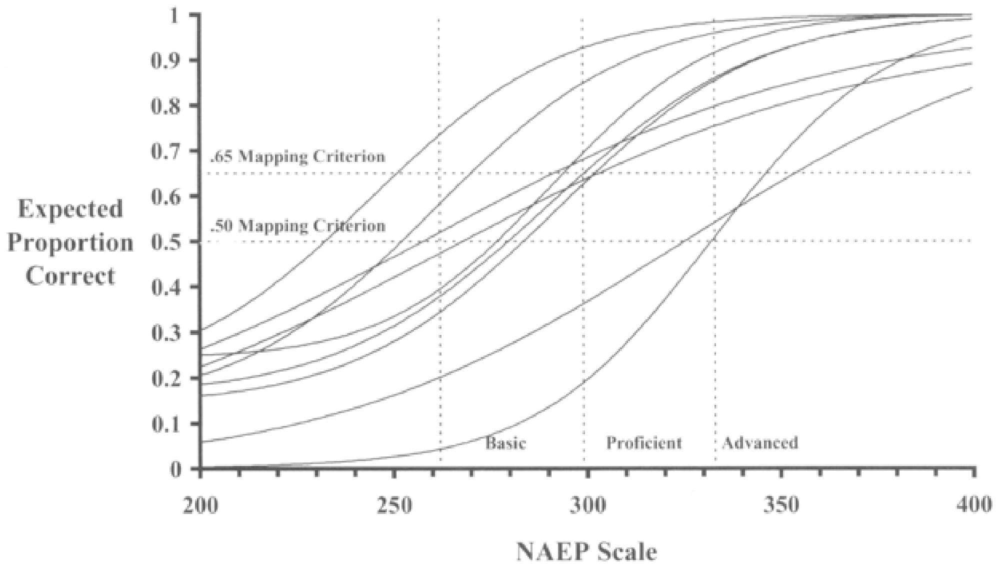
Schulz et al. (2005) created multiple domains within each NAEP content strand through a multistep approach. To begin, curriculum experts worked independently and then together to classify items into domain categories; a panel of teachers also classified items into domains. Final classifications were then determined by a domain classification team that used both sets of substantive classifications, in addition to item-difficulty parameters and teachers' ratings of instructional timing—both with respect to introduction and mastery of item content. Within both Geometry and Data Analysis, three teacher-ordered domains were preserved in the final analysis. However, for the Number Sense, Measurement, and Algebra content strands, greater numbers of teacher domains were collapsed when adjacent categories were overlapping too much in timing and difficulty. Figure 2 from Schulz et al. shows the extent to which individual items "misbehaved" within a single, seemingly homogeneous domain. In contrast, Figure 3 from Schulz et al. shows the more orderly progression of three final Number Sense domains (N1, N2, and N3), constituted as follows from finer grained teacher domains:

N1 Basic Computation with Positive Whole Numbers
Addition and Subtraction of Integers in Context; Rounding and Place Value
Models for Numbers and Operations

N2 Multiplication and Division
Decimals

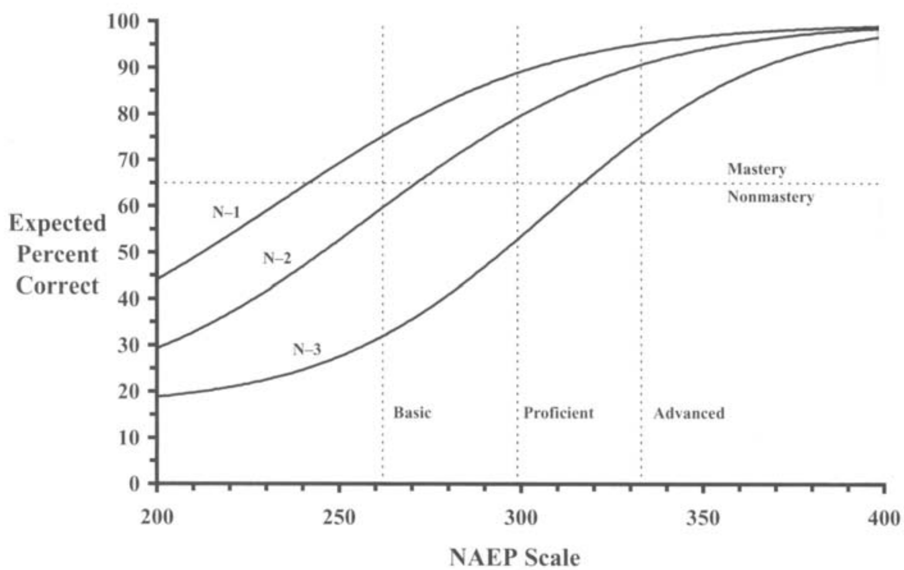
N3 Fractions and Ratios
Rates and Percents
Number Properties
Scientific Notation and Exponents

Figure 2. Item Characteristic Curves in Domain D-2: Uses Graphs and Charts



Source: Schulz et al., 2005. Reprinted with permission from John Wiley and Sons.

Figure 3. Domain Characteristic Curves for Number Sense



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An Illustration of Quasi Learning Progressions for NAEP

Although the demand for curricular neutrality appears to render the use of learning progressions infeasible as a central means for developing NAEP, given the appeal of learning progressions as a way to illuminate the substantive meaning of achievement results, we considered the possibility of constructing "quasi learning progressions" to use as a NAEP reporting device. To do this, we drew on NAEP's anchoring methodology as the psychometric techniques used to locate learning progression tasks and items on a score scale are essentially the same as the anchoring methods used historically by NAEP.

In their guiding document on the construction of progress maps, Masters and Forster (1996) distinguished between "top-down" and "bottom-up" methods for developing learning progressions. Top-down methods involve logically laying out a sequence based on expert judgments about typical pathways for knowledge and skills development. The CCSS and NGSS are examples of top-down methods, except that expert judgments may be strongly grounded in prior experience teaching or studying segments of the progressions. Bottom-up approaches begin and end with empirically gathered evidence and, in this sense, they are essentially norm-referenced approaches. In fact, Masters and Forster (1996) cited NAEP's 1990 Civics Report Card (ETS, 1990) with its item-anchoring method as an example of a bottom-up progress map.

For illustrative purposes, we proposed to construct three hypothetical learning progressions for Graphing, Statistics, and Equations representing two of NAEP's content areas: Data Analysis and Probability, and Algebra. Each of these specific objectives had sufficient numbers of items to make the exercise feasible. We elected to use items and item parameters from NAEP's 2005 eighth- grade mathematics assessment because of our prior work on this particular assessment (Daro et al., 2007; Stancavage et al., 2009) and because most items from the 2005 assessment have subsequently been released.[5] Therefore, it is possible to display various NAEP items illustrating features of the quasi progressions without violating the security of the items. In addition, in the Stancavage et al. study, the Balanced Assessment in Mathematics (BAM; Mathematics Assessment Resource Service, 2002, 2003) was also administered to approximately 2000 examinees and was concurrently scaled and equated to the NAEP scale. As a whole, BAM items were designed to tap higher levels of reasoning and application; therefore, they might be more like the kinds of assessment tasks developed to assess the CCSS.

Using his knowledge of mathematics and research on mathematics learning, study co-author Daro began the development of learning progressions by reviewing all of the items (NAEP and BAM) measuring each of the objectives. Items were ordered on a continuum to represent increasing mastery of the content objective. Items were *not* ordered by perceived difficulty. In particular, items that tapped multiple skills or relied on less familiar formats might be expected to be more difficult for students,

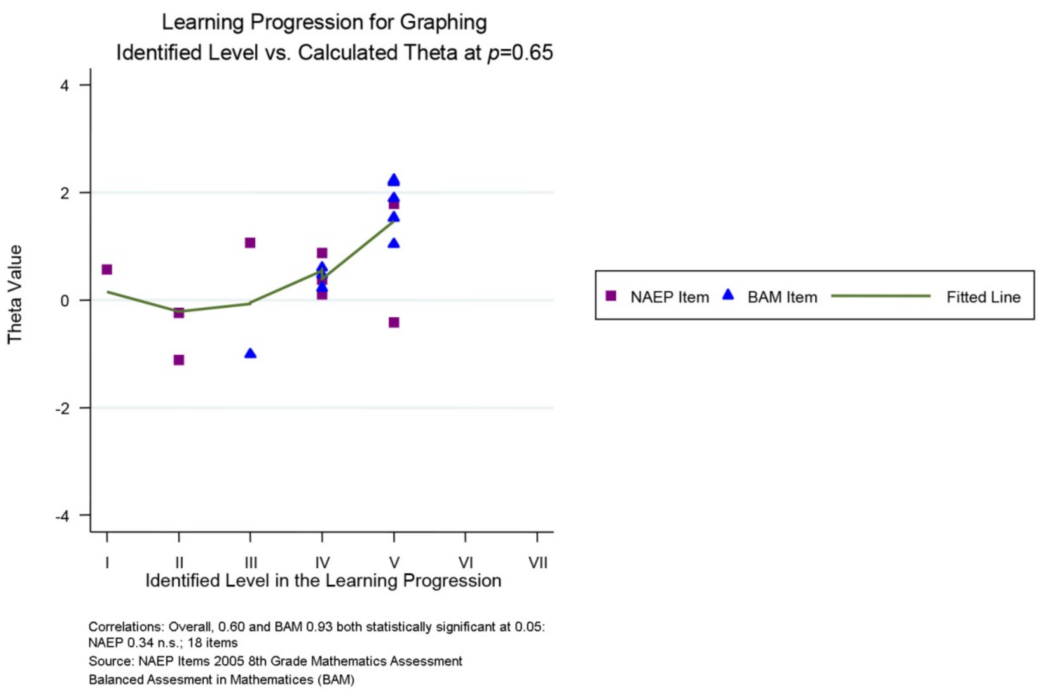
[5]The one item block from the 2005 assessment that has not been released was excluded from our exercise.

but such items were not placed at the higher end of the continuum if they called only for lower level mastery on the objective being rated. The ordering process was conducted following common-sense rules for essay grading and qualitative coding (i.e., only as many distinctions were made as could be reasonably described). Thus, Equations (branch 2) was said to have eight levels but Graphing had only six. Once ordered, items at each level were reviewed by the other authors to confirm that they were similar to each other in terms of the instructional topic addressed, and distinguishable from the next-higher and next-lower levels. Any differences were resolved by discussion among the authors and by using the item descriptors provided by NAEP and BAM, respectively. The items measuring Equations were sufficiently diverse that ultimately two different progressions were created (with some shared items), one calling for the procedural manipulations of equations (branch 1) and the other requiring that students develop equations to represent problem solutions (branch 2).

A critical conceptual decision made by Daro, in consultation with the other study authors, was to order items by the typical instructional sequencing of topics, *not* by cognitive complexity. For example, in statistics, measures of central tendency are usually taught before measures of variability. Very different progressions would have been produced had the ordering dimension been cognitive complexity, but postponing more complex reasoning about subject matter would be antithetical to the intentions of both the CCSS and learning progressions research, which aim to foster greater depth of thinking and reasoning *within* content objectives. For a given topic, of course, instruction usually proceeds from the simplest rendition of a core concept to medium complex and then highly complex understandings and applications of that concept. For two topics, usually taught in the order of A and then B, a highly simplistic ordering might expect to teach and ensure student mastery of all three levels of A before starting with the easiest version of B. In our experience, however, topics are not neatly finished before the next one begins and, in many cases, medium- and high-complexity understandings of any given topic require drawing connections and integrating knowledge and skills from multiple topics. Therefore, for the most part, we kept all items within a given instructional objective at the same level, regardless of whether they were of low, medium, or high complexity. Only when a more advanced application of a topic would typically be taught at a later time was it given a progression level of its own. For example, we created an Equations category called "Inversions" where students were asked to work backwards in applying a rule to a problem situation. Other experts might have argued that these items were just more advanced applications of an earlier level called "Using a rule without formally presenting the equation." We have tried to be as transparent as possible regarding the classification of items so that others may judge how much our findings could change if fundamentally different judgments were made about instructional sequencing. Two NAEP items were eliminated from the Graphing progression because they both involved number line representations that have been controversial with mathematicians. Some BAM items were eliminated or combined with companion items if IRT parameters could not be independently estimated due to the relatively small per-item sample size in the Stancavage et al. (2009) study.

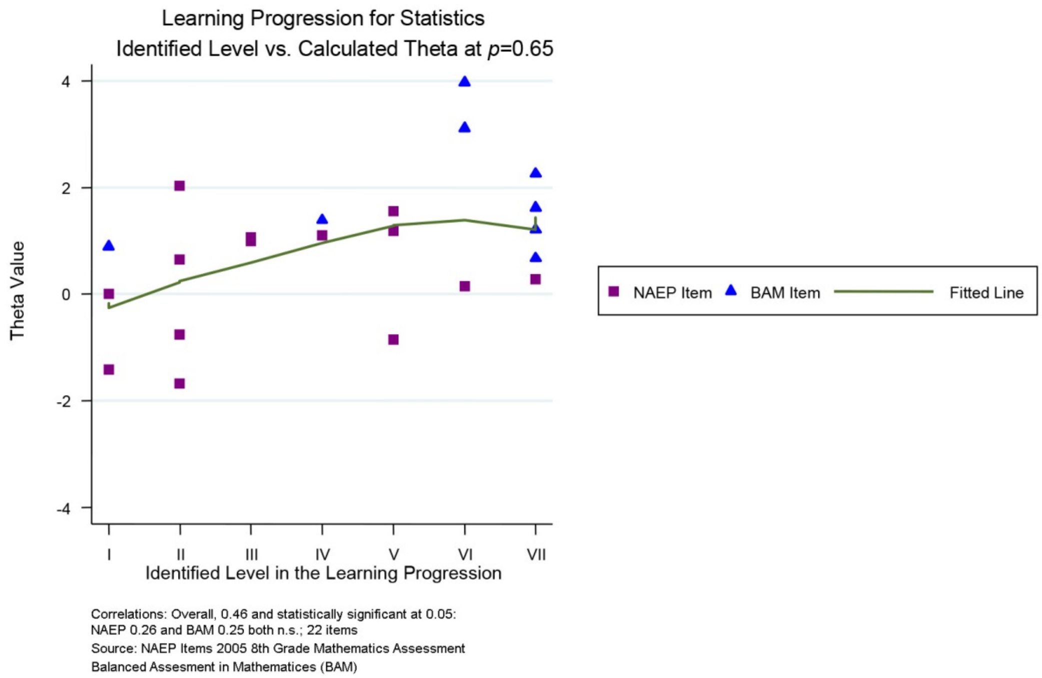
Scatterplots were constructed to provide the simplest portrayal of the relationship between *judged* levels of increasing proficiency on the intended construct and *empirical* evidence of item ordering for each of the four progressions. The *x* axis represents the logically identified levels in the learning progression. The *y* axis represents the empirical value of the items; this empirical value is the value on the IRT score scale (theta value) corresponding to the probability of a correct response of .65 (RP 65). The theta score scale, defining the *y* axis, has a mean of 0 and a standard deviation of 1. Thus an item located at theta = 1 is a relatively difficult item because examinees would need to have a total test score of 1 standard deviation above the mean before they would have a 65 percent chance of getting this item correct. For NAEP items, this scale is the same as the appropriate NAEP subscale for eighth-grade mathematics (e.g., Algebra or Data Analysis and Probability). We have retained the theta metric rather than attempting to convert to a NAEP-like score scale to discourage overinterpretation of individual item locations, especially for BAM items that were calibrated to the NAEP scale using a sample that was not nationally representative. Figures 4–7 are the scatterplots for Graphing, Statistics, Equations branch 1, and Equations branch 2, respectively. Correlations were also computed for each item set overall and separately for NAEP and BAM items.

Figure 4. Scatterplot for Graphing (Theta at RP 65)



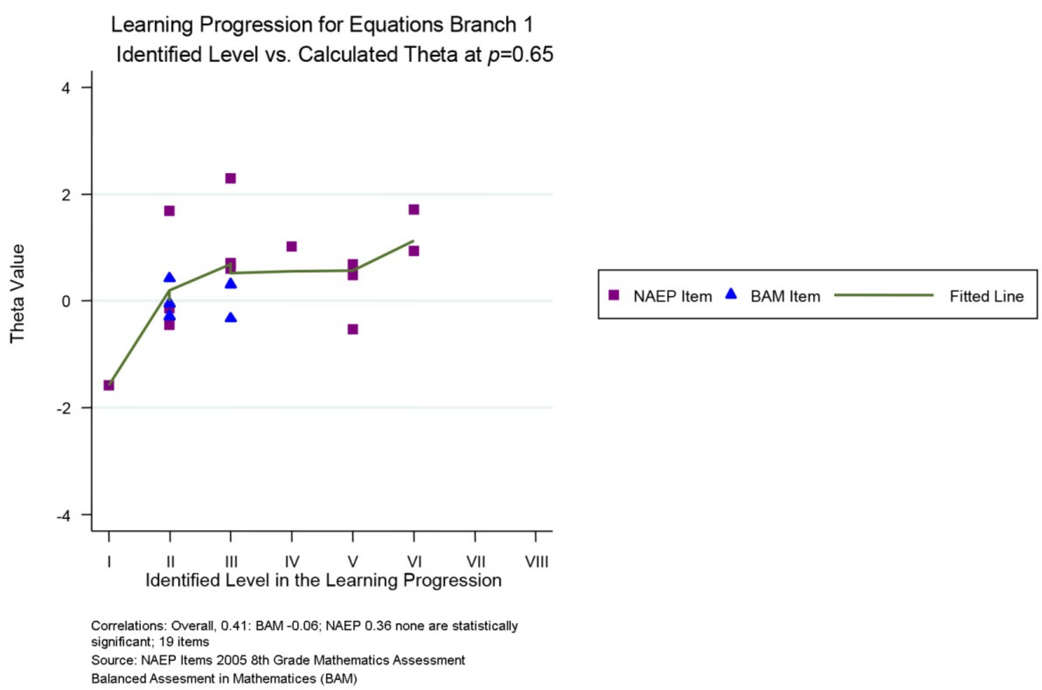
Note: Theta at RP 65=value on the IRT score scale (theta value) corresponding to the probability of a correct response of .65.

Figure 5. Scatterplot for Statistics (Theta at RP 65)



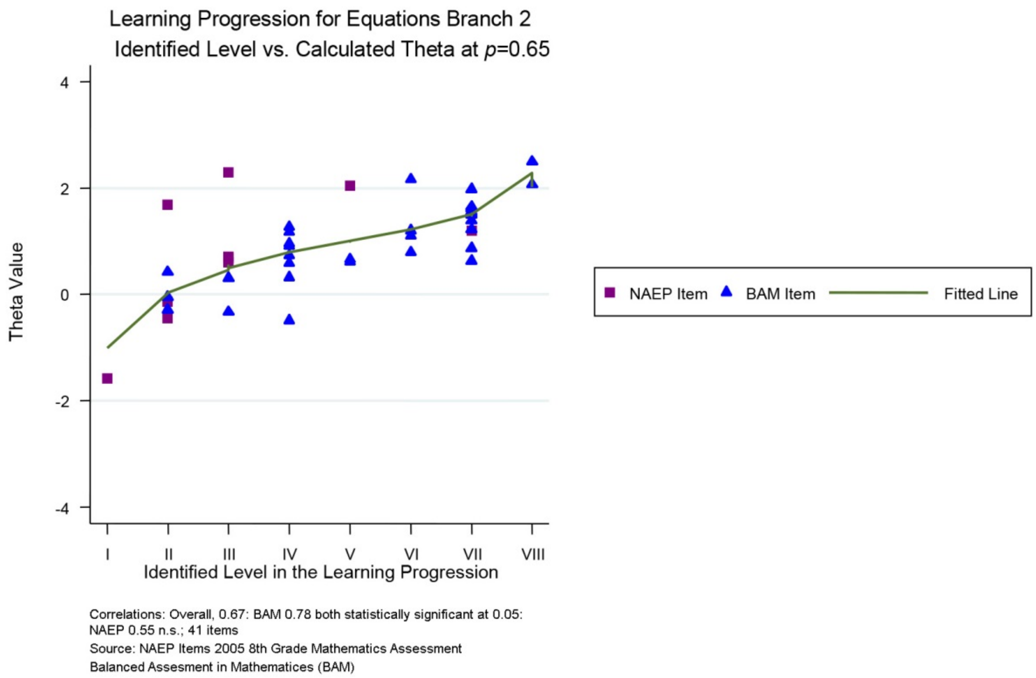
Note: Theta at RP 65=value on the IRT score scale (theta value) corresponding to the probability of a correct response of .65.

Figure 6. Scatterplot for Equations Branch 1 (Theta at RP 65)



Note: Theta at RP 65=value on the IRT score scale (theta value) corresponding to the probability of a correct response of .65.

Figure 7. Scatterplot for Equations Branch 2 (Theta at RP 65)



Note: Theta at RP 65=value on the IRT score scale (theta value) corresponding to the probability of a correct response of .65.

Using the combined NAEP/BAM data sets, the correlation between judged proficiency level and empirical theta was highest (r = .67) for the Equations branch 2 progression, followed by a correlation of .60 for the Graphing progression. The correlations between judged proficiency level and empirical difficulty were somewhat lower for the Statistics and Equations branch 1 progressions, at .46 and .41, respectively. However, even these more moderate correlations suggest that there is indeed a logical and somewhat shared ordering to instructional topics and corresponding student mastery. In general, the combined NAEP and BAM item sets exhibited stronger correlations than either set on its own. In the case of Statistics, combining the item sets improved the degree of relationship from .26 and .25 for the separate item sets to .46 overall. There were very few BAM items assigned to Equations branch 1, but they helped to increase the degree of relationship slightly, from .36 for NAEP items alone to .41 overall. In the case of Graphing and Equations branch 2, however, the logical ordering correlated better with empirical difficulty using BAM items alone rather than in combination with NAEP items.

The vertical spread in these plots illustrates the difficulty in developing assessment items that are so unidimensional that only a single construct determines the level of difficulty. Note also that this vertical spread or range of difficulty within nominally homogeneous groupings of items at each level is nearly identical to the range of difficulty found by Schulz et al. (2005) within domains ordered by instructional timing as illustrated in Figure 2. Several important ideas should be called out to help in interpreting items that are much easier or harder than expected given their location in the logical progression. First, these discrepancies could be caused by *construct-irrelevant variance*, which refers to features of an item that make it hard or easy but have nothing to do with the intended mathematical skill. Typical examples are when excessive verbal demands make an item too difficult for students who actually understand the mathematics, or when item distractors make an item too easy by increasing the possibility of picking the right answer without reasoning through the mathematics. More often, items will be more difficult than expected for the progression level because the mathematical demands are *multidimensional* (i.e., calling for reasoning and connections involving the intended progression construct along with other related mathematical constructs). The interconnecting of graphing skills with mastery of equations is one example. Multidimensionality of assessment items is closely related to our earlier discussion regarding the *degrees of cognitive complexity* within a given progression level. Had we sorted items within a topic category by complexity and moved the more challenging questions later in the progression, we would have reduced the vertical spread and increased the degree of fit between logical and empirical ordering because substantive multidimensionality is often the cause of increased difficulty. In our presentation of each progression, we draw attention to these more challenging and "misfitting" items, and encourage the reader to consider whether they are misplaced. Again, our argument is that to move such items higher in the progression would mean that the intention of the instructional sequence is to postpone reasoning and depth of understanding.

The issue of multidimensionality is also closely related to the issue of *curriculum-specificity*. Although orderings are usually widely shared within very narrow skill domains (e.g., adding fractions with like denominators always comes before unlike denominators), combining domains is usually an arbitrary decision made uniquely by each separate curriculum. For example, relating formulae and graphs comes much later in some curricula than others. We should also acknowledge that the apparent misfit in the scatterplots could be due to conceptual *inaccuracies* in our assignment of items to levels.

In our discussion of each progression, we refer to these types of explanations for within-level variations in item difficulty. Note that, for instructional purposes, within-level variation (from easiest to most challenging) could describe the sequencing of reasoning and deepening of understanding within a given unit of instruction, whereas the left-to-right sequencing of levels could describe the longer term ordering of concepts to be mastered over the course of many years of study. These two different orderings, within and across levels, are necessitated by the framing of this exercise in terms of the CCSS and the effort to represent mastery over broad reach of content. By contrast, Sztajn, Confrey, Wilson, and Edgington (2012), citing research on task analysis and discourse practices, argue that learning trajectories can guide teachers in responding to student thinking even within a single lesson focused on a specific task, but always with attention to the long-term goals of "fostering higher levels of sophistication over time" (p. 150). Although in many cases, we can make sense of the vertical spread instructionally, this heterogeneity illustrates the problem of using learning progressions to anchor the NAEP scale. The natural tendency would be to use the middle items that best fit the progression to anchor and describe the score scale, but for examinees scoring at any given score level this would ignore the complex items at that level that they cannot do as well as the easier items at higher levels that they can do.

Graphing Learning Progression

Data for the Graphing learning progression are presented in Table 3, while the text of the items is shown in Appendix Figure A1. Level I is represented by only one item, which asks students to follow directions to extend a pattern on a grid. The graphic knowledge involved is extremely simple, but the item has a higher than expected theta value (0.57), most likely because of the verbal demands of the item. Two items were classified as Level II items. Both involve locating a point on a grid and are relatively easy, with thetas of -1.12 and -0.24, respectively, although one can see the instructional progression from finding an intersection of number and letter dimensions on a map to formal coordinates. Level III items represent a slightly higher increment over Level II in that students must now determine an answer by locating the correct point on a curve that satisfies the problems' conditions. The first of these, A Swimming Race-item 2, which involves finding how long the winner took to swim the 50-meter race, is very easy (theta = -1.01). (Note that the full set of BAM items is shown in the figure, even though questions 3, 4, and 5, are discussed later in the progression.) By contrast, the second Level III item is quite challenging (theta = 1.06), presumably because eighth-grade students have not had experience estimating the value of a point on a curve that does not pass through a whole-number location on the grid. This could be thought of as an example of multidimensionality and/or curriculum specificity in that students would typically not be exposed to this type of question until much later in the curriculum, in the context of functions. However, the point estimation idea could be taught independent of functions, and this curricular decision would affect the fit of this item with Level III of the progression.

Items in Level IV Graphing all represent a greater knowledge of linear relationships and use of the coordinate system compared with Levels II and III. Theta locations range from 0.10 to 0.87. Level V items are a more significant step up, for the first time clearly linking Algebra and Graphing by asking students to relate linear formulas and graphs. With the exception of the first item in the level, all Level V items are quite difficult, requiring that students be 1.5 to 2 standard deviations above the mean before they have a 65 percent chance of getting the item correct. The first item is easier due to the instructions that tell students how to find the answer: "Graph the five points that represent the savings on the grid below and connect the points with a dotted line." Our observation that Level IV represents a small conceptual increment over prior levels, whereas Level V is a more significant step is consistent with the ordinal nature of the levels. No claim is made that these judgments represent an equal interval scale.

Table 3. Judged Levels and RP 65 Theta Locations for the Graphing Learning Progression

Item Identifier	Level	Theta	Level Description
XH000442	I	0.57	Follow directions to draw a line graph
VB335166	II	-1.12	Locate a point on a grid
VB434925	II	-0.24	
A Swimming Race-Item 2	III	-1.01	In a grid, locate a point on a curve
YJ000078	III	1.06	
VB429681	IV	0.1	Using lines to describe trends, find points
Vacations-Item 1	IV	0.23	
AP000711	IV	0.38	
Dollars-Item 1	IV	0.47	
Dollars-Item 2	IV	0.60	
VB434830	IV	0.87	
YJ000089	V	-0.42	Relate linear formula to graph
Dollars-Item 3	V	1.04	
Party-Item 5	V	1.53	
VB434934	V	1.79	
A Swimming Race-Item 4	V	1.89	
A Swimming Race-Item 5	V	2.19	
A Swimming Race-Item 3	V	2.23	

Note: RP 65 = value on the IRT score scale (theta value) corresponding to the probability of a correct response of .65

Statistics Learning Progression

Items for a possible Statistics learning progression are presented in Appendix Figure A2 with corresponding data shown in Table 4. Level I items require simple reading of information from graphical displays. The first item is correspondingly very easy (theta = -1.42). The next item is similar in terms of the mathematics elicited, but is much more difficult because of the demand characteristics of the item's format and language. Boxes of Candy item 2 (theta = 0.89) is an example of difficulty possibly due to curriculum specificity. It is conceptually simple for adults but could be difficult for eighth graders who might not yet have been taught about reading this type of information from bivariate plots. Level II items represent a step up from Level I items, asking students to produce a graph or describe relationships by extracting multiple pieces of information from graphs. Items in Level II vary tremendously in difficulty, from theta = -1.68 to 2.03, illustrating how much the particular demand characteristics of items affect the conclusion: "Yes, this student can interpret information from graphs."

Items addressing measures of central tendency comprise Level III. These items are relatively difficult, ranging from theta = 0.99 to 1.07. Level IV items tap more advanced understandings of central tendency. All three items are difficult, but the third item, which asked students to explain their reasoning for picking the median over the mean to represent the typical number of customers at Malcolm's Bike Shop over a five-day period, was almost impossibly difficult (theta = 7.62). Level V content returns to graphical interpretation and includes items that clearly would have been taught later than Level II graphical interpretation content, but note that how much later varies from one curriculum to the next. Level VI items test students' knowledge of sampling and variation, with theta values ranging from 0.15 to 3.97. Level VII items assess students' ability to interpret scatterplots and their use of sampling strategies to estimate large numbers. Theta values ranged from 0.28 to 2.26.

Table 4. Judged Levels and RP 65 Theta Locations for the Statistics Learning Progression

Item Identifier	Level	Theta	Level Description
VB335159	I	-1.42	Read from a graphical representation
HW000854	I	0.01	
Boxes of Candy-Item 2	I	0.89	
IY002250	II	-1.68	Interpret from a graphical representation
OM000557	II	-0.76	
YJ000102	II	0.65	
YJ000093	II	2.03	
VB335157	III	0.99	Measures of central tendency
VB434825	III	1.07	
IY002422	IV	1.1	Advanced measures of central tendency
Ages-Item 3	IV	1.39	
HL002246	IV	7.62	
VB417888	V	-0.86	Advanced graphical interpretation
VB434849	V	1.18	
YJ000060	V	1.56	
AP000506	VI	0.15	Indicators of variance
Best Guess-Item 2	VI	3.11	
Best Guess-Item 1	VI	3.97	
VB417891	VII	0.28	Measures of correlation and Estimation
Bacteria-Item 1	VII	0.67	
Boxes of Candy-Item 4	VII	1.21	
Bacteria-Item 2	VII	1.62	
Bacteria-Item 3	VII	2.26	

Note: RP 65 = value on the IRT score scale (theta value) corresponding to the probability of a correct response of .65

Equations Learning Progression Branch 1

The items measuring Equations were sufficiently diverse that ultimately two different progressions were created, one calling for procedural manipulations of equations (branch 1) and the other requiring that students develop equations to represent problem solutions (branch 2). Branch 1 appears in Table 5 and Appendix Figure A3, while branch 2 appears in Table 6 and Appendix Figure A4.

The first three levels of these two progressions are the same, but they separate into two distinct branches at Level IV. Level 1 is represented by a single item. It is an elementary-level, prealgebra item that asks students to figure out the missing value in a simple number sentence. Consistent with its judged level in the progression, the item also has a very easy theta value of -1.58. Level II asks students to evaluate an expression for a specific value or to complete a pattern by simple recursion. For example, in the first Apartment Numbers problem, students can complete the pattern by counting. In Boxes of Chocolates, the pictures help them see whether to "add two each time" or "add three each time." More advanced find-the-rule or develop-a-formula problems occur in later levels of the Equations learning progression branch 2. Theta values for Level II range from -0.45 to 0.42. This range excludes the last item in Level II, which we judged to be unusually difficult (theta = 1.69), due to construct irrelevant variance associated with format and linguistic demands. Items in Level III ask students to find and use an algebraic formula. They do not have to develop a formal equation, only recognize appropriate expressions. Theta values range from -0.33 to 0.71, with the exception of the final item, which has a theta value of 2.30. This last item is a bit odd as a test of algebra understanding and might better be used as a classroom activity to introduce the concept of slope.

Level IV has only one item and might therefore be combined with the next higher level, although we can imagine other similar items that test students' understandings of basic algebraic principles—in this case an understanding of the distributive property. This item is clearly more difficult than preceding levels (theta = 1.02), but is also more difficult than items in the subsequent level. Level V items ask students to manipulate equations, solving for x, or to identify equivalent expressions. Theta values range from -0.53 to 0.69. The last level in branch 1 asks students to use a formula to solve a problem. Problems of this type are more typically introduced as students begin formally working with functions. Correspondingly, the items are more difficult for students, with theta values of 0.93 and 1.71.

Table 5. Judged Levels and RP 65 Theta Locations for the Equations Learning Progression Branch 1

Item Identifier	Level	Theta	Level Description
HL000844*	I	-1.58	Supply the missing number
VB417883*	II	-0.45	Evaluate an expression for a specific value Determine an expression to model a scenario
Tilling Squares-Item 1*	II	-0.29	
VB434929*	II	-0.14	
Apartment Numbers-Item 1	II	-0.05	
Boxes of Chocolates-Item 1*	II	0.42	
EL001490*	II	1.69	
Emma's Models-Item1 *	III	-0.33	Determine equations Linear relationship between two quantities
Party-Item 1*	III	0.31	
VB335172*	III	0.60	
VB434848*	III	0.68	
VB335163*	III	0.71	
XH000443*	III	2.30	
VB335154	IV	1.02	Identify an equivalent algebraic expression
YJ000107	V	-0.53	Represent a quantitative relationship with an equation Solve for an algebraic equation
VB335169	V	0.48	
AP000710	V	0.69	
VB434852	VI	0.93	Functions
HW000857	VI	1.71	

Note: RP 65 = value on the IRT score scale (theta value) corresponding to the probability of a correct response of .65
*Same as Branch 2

Equations Learning Progression Branch 2

The two Equations progressions share the first three levels. All eight levels of branch 2 are shown in Table 6. Here we describe the unique levels of the second branch, beginning with Level IV. Although earlier levels required students to recognize and extend a number pattern, Level IV items require development of rules (rather than selecting a rule) and/or more significant extensions. The easiest item in this level—with a theta value of -0.49—asks for an extension of the pattern to the top apartment in the 10th house. The most difficult item (theta = 1.27) is also an extension of a pattern, but adds the challenge of understanding the geometry of the situation in order to calculate the number of white tiles that must be added each time. Items in Level V are quite similar to those in Level IV except that students must also explain their reasoning (i.e., they must give a verbal description of the pattern or rule). Items at Level VI also are similar to Level IV problems except that students are asked to invert their understanding of the rule—a slightly more complex task and one that would typically come after instruction focused on generating a rule and explain one's thinking about a pattern or rule. Note that none of these imply that instruction on one level is finished before moving on to the next, but we have tried to represent the sequencing of how these levels are typically introduced and perhaps how they might eventually be mastered. Items in Level VII go further and ask students to develop a formal expression for their conceptual rule. Although a few items at Level VII are easier than Level IV, as a set they are substantially more difficult, illustrating the important conceptual step required to move from pattern describing to formal algebraic representation. The two items in Level VIII ask students to conceptualize and relate two rules to find the problem solution. This last type of problem would be used to introduce and motivate the need for solving systems of equations.

Table 6. Judged Levels and RP 65 Theta Locations for the Equations Learning Progression Branch 2

Item Identifier	Level	Theta	Level Description
HL000844*	I	-1.58	Supply the missing number
VB417883*	II	-0.45	Evaluate an expression for a specific value Determine an expression to model a scenario
Tilling Squares-Item 1*	II	-0.29	
VB434929*	II	-0.14	
Apartment Numbers-Item 1	II	-0.05	
Boxes of Chocolates-Item 1*	II	0.42	
EL001490*	II	1.69	
Emma’s Models-Item 1*	III	-0.33	Determine equations Linear relationship between two quantities
Party-Item 1*	III	0.31	
VB335172*	III	0.60	
VB434848*	III	0.68	
VB335163*	III	0.71	
XH000443*	III	2.30	
Apartment Numbers-Item 2	IV	-0.49	Use a rule without formally presenting the equation
Cups-Item 5	IV	0.32	
Fish Ponds-Item 2	IV	0.59	
Party-Item 2	IV	0.73	
Design a Garden-Item 3	IV	0.91	
Cups-Item 3	IV	0.96	
Cups-Item 2	IV	1.17	
Tiling Squares-Item 2	IV	1.27	
Fish Ponds-Item 3	V	0.61	Explain reasoning
Vacations-item 3	V	0.66	
VB434859	V	2.05	
Fish Ponds-Item 4	VI	0.79	Inversions
Apartment Numbers-Item 3	VI	1.10	
Party-Item 4	VI	1.20	
Design a Garden-Item 4	VI	2.16	
Emma’s Models-Item 4	VII	0.63	Develop a formal expression
Fish Ponds-Item 5	VII	0.87	
EL001486	VII	1.20	
Fish Ponds-Item 6	VII	1.22	
Apartment Numbers-Item 4	VII	1.39	

Item Identifier	Level	Theta	Level Description
Tiling Squares-Item 4	VII	1.50	
Apartment Numbers-Item 5	VII	1.52	
Tiling Squares-Item 3	VII	1.56	
Tiling Squares-Item 5	VII	1.62	
Cups-Item 6	VII	1.65	
Party-Item 3	VII	1.98	
Cups-Item 7	VIII	2.07	System of two equations
Picking Apples-Item 3	VIII	2.50	

Note: RP 65 = value on the IRT score scale (theta value) corresponding to the probability of a correct response of .65
*Same as Branch 1

Conclusions

Learning progressions are a highly popular innovation in assessment and instructional design. The core principles of learning progressions have strong theoretical and research grounding, although specific, practical instantiations are rare, at least in U.S. contexts. Given the salience of hypothesized learning progressions in the design of the CCSS and NGSS, it is important to consider the relevance of formally developed learning progressions for the future design of NAEP.

The CCSS and NGSS are narrative documents, similar to past standards documents, and, as such, are likely to influence the crafting of the next NAEP frameworks in a variety of ways. In this paper we considered the relevance of more *formally developed learning progressions* for NAEP, which would involve more detailed development of instructional activities and corresponding assessment tasks tied to the frameworks. Because NAEP must be sufficiently robust to assess progress on the standards across multiple curricula (unlike assessments in countries with a single, national curriculum), it is highly unlikely that formal learning progressions could be the main building blocks of a newly design NAEP. Furthermore, even if the intention were to create Grade 4 and Grade 8 cross-sections for NAEP that are consistent with CCSS sequences, it is important to recognize that more formal progressions at the needed level of specificity do not yet exist, and developing and field testing progressions is a much more extensive and costly procedure than assessment design alone.

If curriculum-linked learning progressions cannot be the primary or central building blocks for NAEP, the assessment must nonetheless be designed in such a way as to monitor the success of deeper curricular reforms where they occur. To continue to be an independent monitor and even a check on other assessments, NAEP must have a strategic vision that attends to both breadth and depth in representing subject-matter expertise. In a recent white paper on the future of NAEP (National Center for Education Statistics, 2012), an expert panel recommended that NAEP domain specifications be broadened so as to enable linkages with multiple other assessments, including long-term trend versions of NAEP, international assessments, and state consortium assessments. *Under such a design, the NAEP framework and reporting domain need not be the same as this comprehensive item pool, which might be thought of as a "super-assessment" domain or blueprint.* Until now, a NAEP framework has always been used as the complete blueprint for the intended assessment. Items were developed to represent the framework, and performance was reported in terms of the intended framework. In contrast, the 2012 panel recommended a dynamic approach to constituting the content domain of NAEP administrations so as to address explicitly how changing definitions of subject-matter domains affect immediate outcomes and reports of progress over time. More specifically, the NAEP reporting framework as historically conceived would be situated within a larger, super-assessment domain. Like a series of Venn diagrams, other assessment domains would also be located within the super assessment, with carefully designed shared and unique item sets. By spiraling these various assessments together in a single NAEP administration, the means for linking and equating studies would be built in rather than requiring separate linking studies.

The panel also cautioned that NAEP may not be able to administer its most ambitious and innovative assessment tasks to random samples of students because a lack of opportunity to learn could make the assessment too difficult for the majority of students. Instead, the panel recommended that NAEP first conduct special studies, as have been undertaken in the past, to determine whether more advanced performance can be documented in those settings where reform curricula have been successfully implemented. Thus, assessment tasks tied to learning progressions in mathematics, science, or literacy could be embedded within the NAEP super-assessment framework, and both performance outcomes and the psychometric functioning of the assessment tasks could be compared for students with and without instructional opportunities tied directly to learning progressions curricula.

In this study, we used familiar anchoring methodology to construct four quasi learning progressions from existing NAEP items in combination with BAM items. This exercise allowed us to consider the feasibility of building example learning progressions into the NAEP item pool to enable their use as a reporting strategy. Based on this exercise, we conclude that such an approach is infeasible and likely to be misleading until there is more widespread implementation of new standards and thereby greater congruence between hoped-for and empirical ordering of items. Although we can see ways to improve the meaningfulness of quasi learning progressions by eliminating misfitting items, in most cases these are not items that one would want to remove lightly. In the case of items found to be unpredictably difficult because of construct irrelevant variance, removing the items would have an overall positive effect on assessment quality. However, this particular reason for misfitting items occurred relatively rarely. The more difficult problem has to do with items that did not fit the intended progression because of cognitive challenges often caused by multidimensionality and/or curriculum specificity that might not be as misfitting if students had more direct experience with this type of item. Such items should not be eliminated from the assessment because they represent the very ambitions of the new standards documents. To anchor the scale with only the well-behaved items essentially moves more challenging items to a later place on the progression. These kinds of decisions can only be made after doing the kind of work that is required for the development of learning progressions (i.e., logical and expert-developed sequences must be tested in instructional contexts where students have had the opportunity to learn with the support of curricula specifically developed in conjunction with the intended progression).

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Appendix A. Items in Learning Progressions

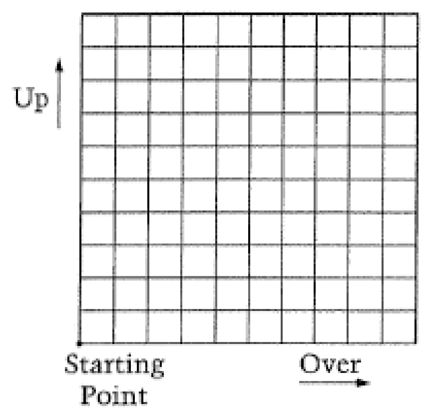
Figure A-1. Graphing Learning Progression

Level I

10. From the starting point on the grid below, a beetle moved in the following way. It moved 1 block up and then 2 blocks over, and then continued to repeat this pattern. Draw lines to show the path the beetle took to reach the right side of the grid.

Level I
Theta 0.57

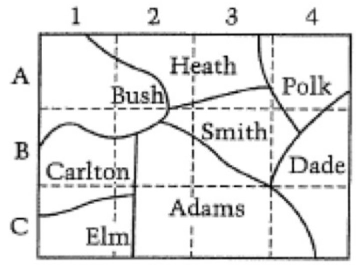
XH000442



Item XH000442.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z12M4B.

Level II



Level II
Theta -1.12

14. The map above shows eight of the counties in a state. The largest city in the state can be found at location B-3. In which county could this city lie?

Ⓐ Adams or Carlton

Ⓑ Adams or Smith

Ⓒ Carlton or Elm

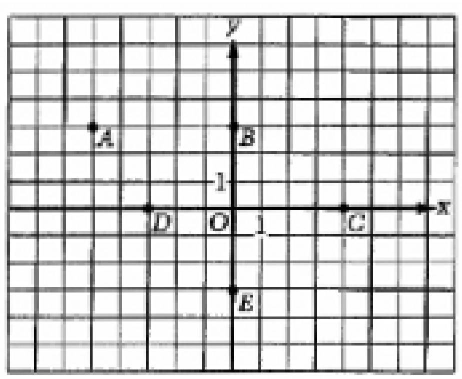
Ⓓ Dade or Polk

Ⓔ Polk or Smith

VB335166

Item VB335166.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z2M12.



2. The graph above shows lettered points in an (x, y) coordinate system. Which lettered point has coordinates $(-3, 0)$?

Ⓐ *A*

Ⓑ *B*

Ⓒ *C*

Ⓓ *D*

Ⓔ *E*

Level II
Theta -0.24

VB434925

Item VB434925.

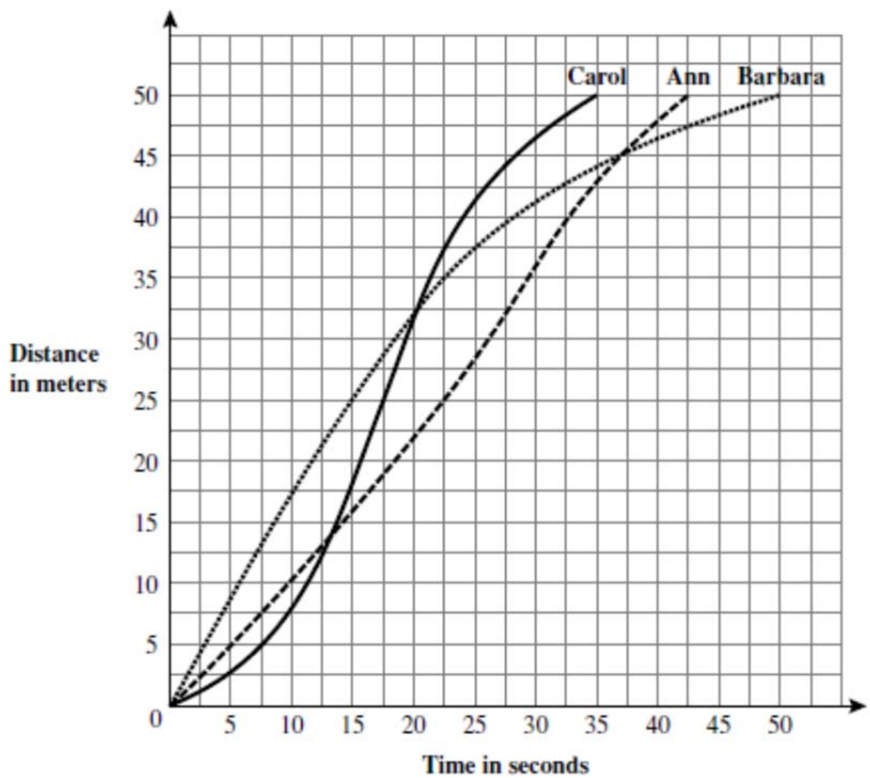
Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z2M11**.**

Level III

A Swimming Race

This problem gives you the chance to:
- describe a race, given a distance-time graph

Ann, Barbara and Carol decided to have a race in the swimming pool.
This graph shows what happened during the 50-meter race.
The lines labeled Ann, Barbara, and Carol show the distances from the starting point for the three swimmers at different times during the race.



1. Who was the winner? ____________

2. How long did the winner take to swim the 50-meter race? ____________ seconds

Level III
Theta -1.01

Imagine you are the radio commentator for the race.
Describe what is happening to each of the competitors during each stage of the race.

3. Stage One: 0–15 seconds

Level V
Theta 2.23

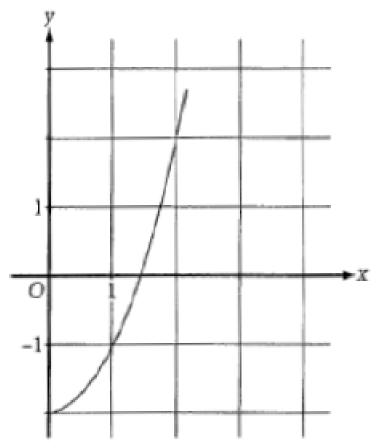
4. Stage Two: 15–30 seconds

Level V
Theta 1.89

5. Stage Three: 30–50 seconds

Level V
Theta 2.19

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.



11. On the curve above, what is the best estimate of the value of x when $y = 0$?

Ⓐ −2.0

Ⓑ 1.1

Ⓒ 1.4

Ⓓ 1.7

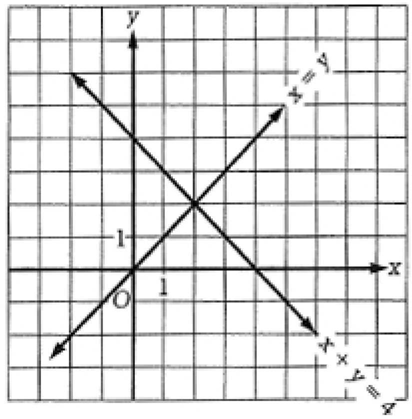
Ⓔ 1.9

Level III
Theta 1.06

Item YJ000078.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z12M3B.

Level IV



7. Which point is the solution to both equations shown on the graph above?

Ⓐ (0, 0)

Ⓑ (0, 4)

Ⓒ (1, 1)

Ⓓ (2, 2)

Ⓔ (4, 0)

Level IV
Theta 0.1

Item VB429681.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z12M3B.

Vacations

This problem gives you the chance to:
- analyze relationships using graphs and algebra

Here is some information about how some students are paying for their summer vacations.

Carla: Her mom gave her $100 in January and Carla has saved $25 every month since, starting in February.

Arnie: Arnie put $150 in his piggy bank in January.

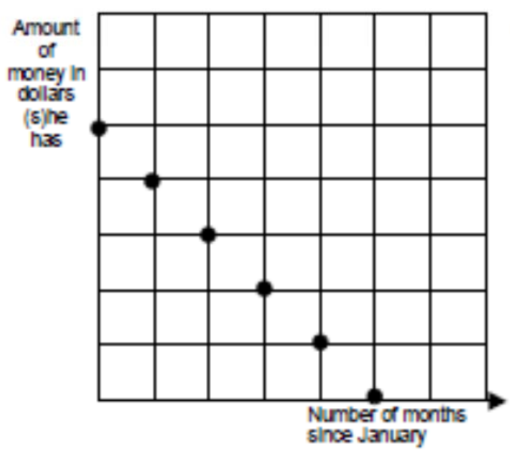
Sue: Sue booked her vacation in January. She had $250 in her piggy bank. Starting in February, she is paying $50 each month to the travel company.

Ben: Starting in February, Ben saves $30 every month.

Here are some graphs illustrating these situations.

Level IV
Theta 0.23

1. Match each person with a graph and explain how you decided.

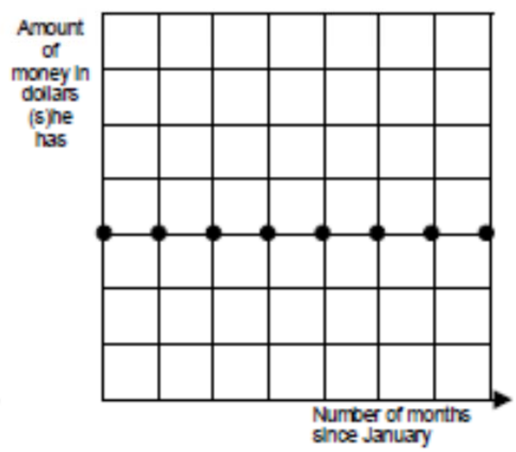


Name: ____________________

Reason: ____________________

____

____

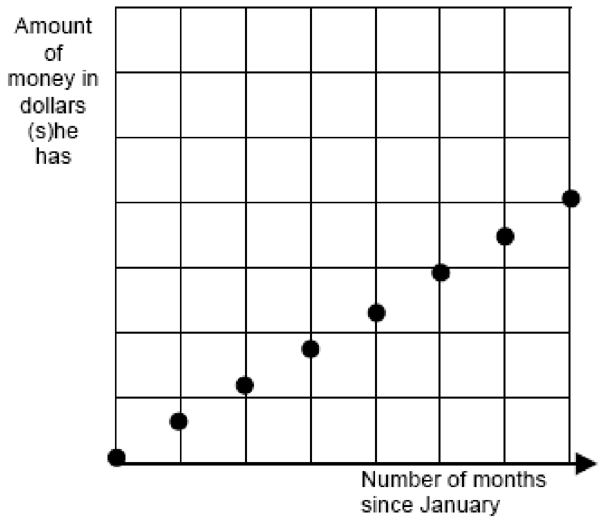


Name: ____________________

Reason: ____________________

____

____

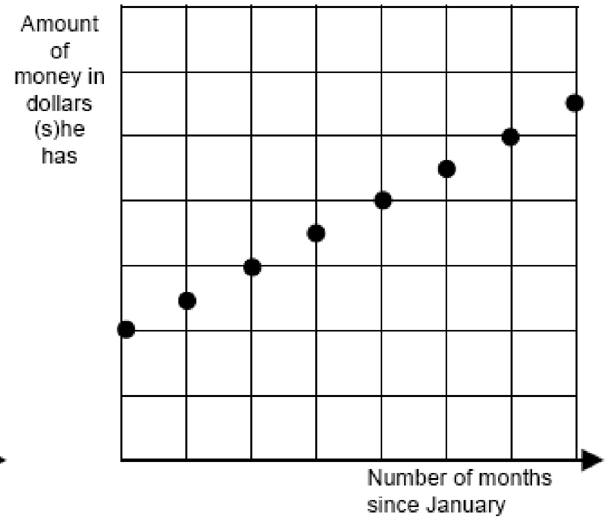


Name: ____________________

Reason: ____________________________

____

____



Name: ____________________

Reason: ____________________________

____

____

2. In these equations, $\$A$ is the amount of money and n is the number of months since January.

   $A = 250 - 50n$
   $A = 30n$
   $A = 150$

   a. Find the person for each of these equations.
   b. Write a formula for the fourth person.

   Carla ______________________________

   Arnie ______________________________

   Sue ________________________________

   Ben ________________________________

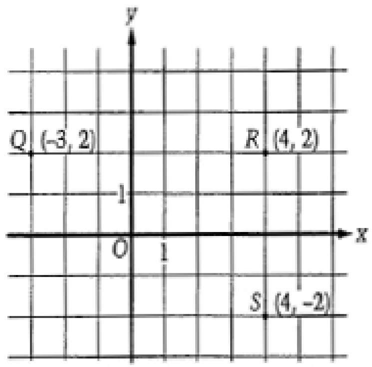
3. Write a possible description for this formula: $A = 50n + 150$

   ____

   ____

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Note: Only Item 1 from Vacations pertains to this progression. The remaining items 2 and 3 do not occur in this progression.



13. If the points Q, R, and S shown above are three of the vertices of rectangle $QRST$, which of the following are the coordinates of T (not shown) ?

Level IV
Theta 0.38

AP000711

Ⓐ (4, − 3)

Ⓑ (3, − 2)

Ⓒ (−3, 4)

Ⓓ (− 3 , − 2)

Ⓔ (−2, − 3)

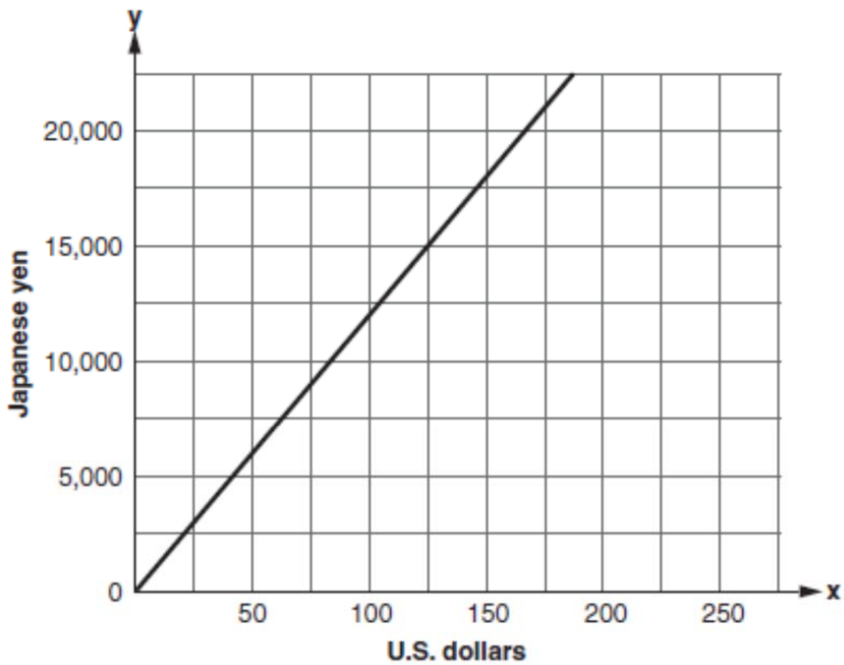
Item AP000711.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8 Block Z12M3B.

Dollars

This problem gives you the chance to:
- use a graph to convert currency

This graph can be used to convert between U.S. dollars and Japanese yen.



1. Use the graph to estimate how many Japanese yen you would get for 100 U.S. dollars.

   On the graph, show how you found your answer. ________________

Level IV
Theta 0.47

2. Use the graph to find out how many U.S. dollars you would get for 20,000 Japanese yen.

   Show how you found your answer. ________________

Level IV
Theta 0.60

3. Use the graph to estimate the number of Japanese yen you would get for 1,000 U.S. dollars.
   Explain how you figured it out. ________________

   ____

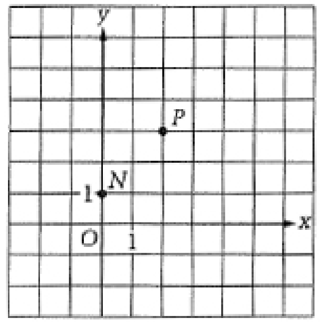
   ____

   ____

   ____

Level V
Theta 1.04

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.



VB434830

14. For the figure above, which of the following points would be on the line that passes through points N and P ?

Level IV
Theta 0.87

Ⓐ $(-2, 0)$

Ⓑ $(0, 0)$

Ⓒ $(1, 1)$

Ⓓ $(4, 5)$

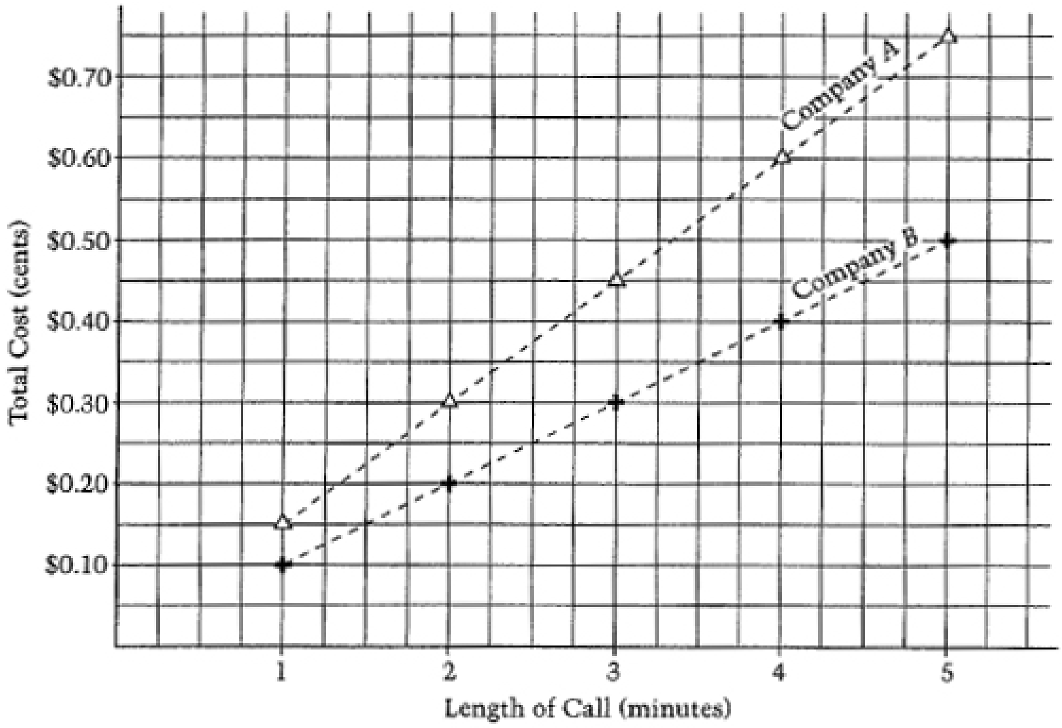
Ⓔ $(5, 4)$

Item VB434830.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block B2M10.

Level V

20. The graph below shows the cost that two long-distance telephone companies each charge for calls of various lengths (in minutes).



a. What is the cost of a 4-minute call using Company *B* ?

____

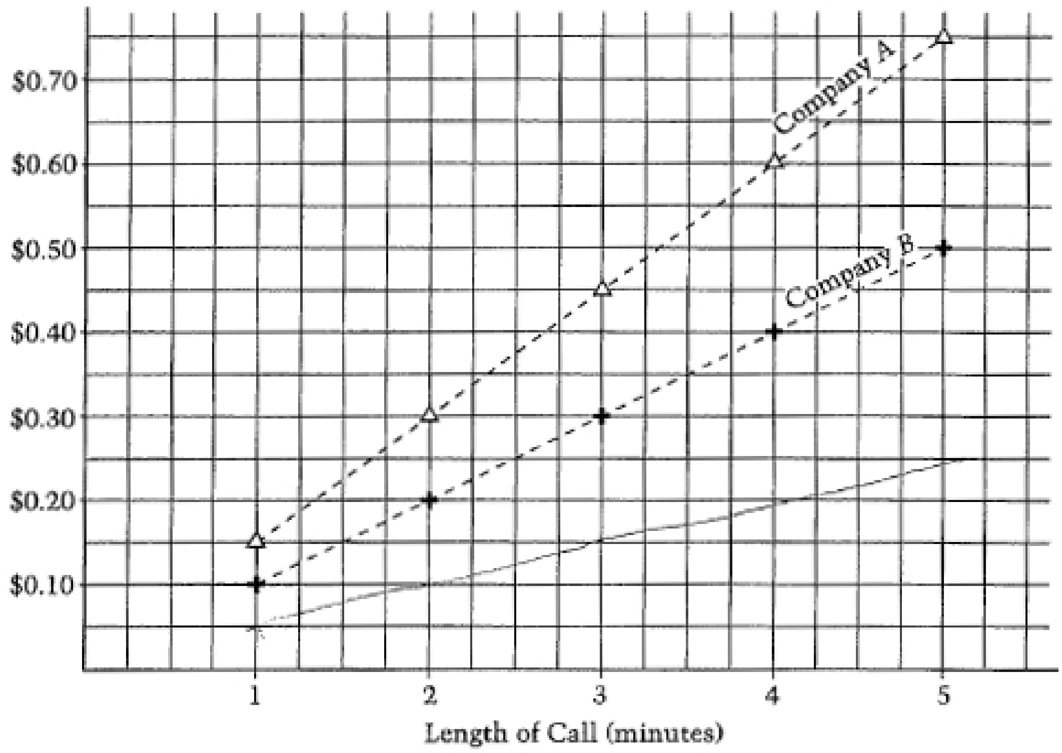
b. What is the cost per minute for a call using Company *B* ?

____

c. Determine the amounts of money saved (in cents) by using Company *B* instead of Company *A* when calls of 1, 2, 3, 4, and 5 minutes are made. Then graph the five points that represent the savings on the grid below and connect the points with a dotted line.

Level V
Theta -0.42

YJ000089



Item YJ000089.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z12M4B.

For Dollars Item 3 (Level V, Theta 1.04), please see page 188.

Party

This problem gives you the chance to:

- choose and use number operations in context
- find and use an algebraic formula
- relate formulae and graphs

Sarah is organizing a party at the Vine House Hotel.

1. Sarah thinks there will be 60 people at the party.
Show that the cost will be $1350.

2. What is the cost of a party for 100 people at the Vine House Hotel? $ ____________
Show how you figured it out.

3. *C* dollars is the cost of a party for *P* people.
Find a formula that gives *C* in terms of *P*.

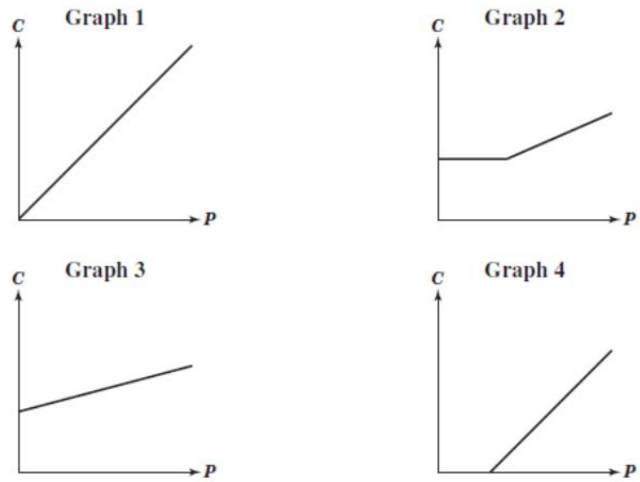
____

____

4. Sarah's party cost $1750 in all.
How many people came to the party? ____________
Show your calculations.

5. Which of these graphs shows the connection between the number of people at the party, *P*, and the cost, *C*?

Level V
Theta 1.53



Explain how you figured it out.

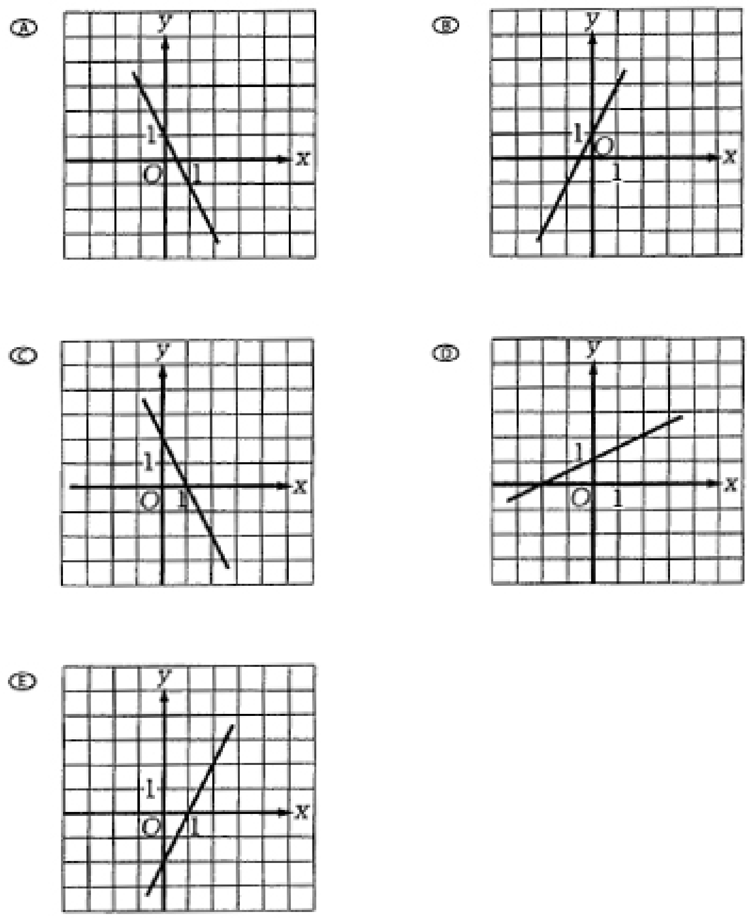
Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Note: Only Item 5 from Party pertains to this progression. The remaining items 1, 2, 3, and 4 do not occur in this progression.

11. Which of the following is the graph of the line with equation $y = -2x + 1$?

Level V
Theta 1.79

VB434934



Item VB434934.

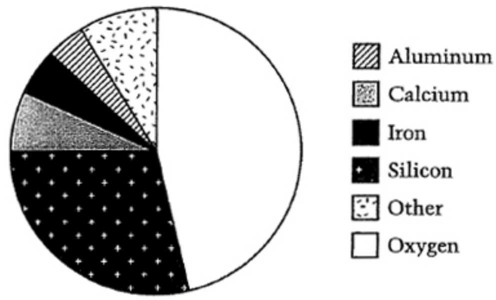
Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z2M11.

For A Swimming Race Item 4 (Level V, Theta 1.89), please see page 182.
For A Swimming Race Item 5 (Level V, Theta 2.19), please see page 182.
For A Swimming Race Item 3 (Level V, Theta 2.23), please see page 182.

Figure A-2. Statistics Learning Progression

Level I

ELEMENTS THAT MAKE UP THE EARTH'S CRUST



8. According to the graph above, which element forms the second greatest portion of the earth's crust?

Ⓐ Oxygen

Ⓑ Silicon

Ⓒ Aluminum

Ⓓ Iron

Ⓔ Calcium

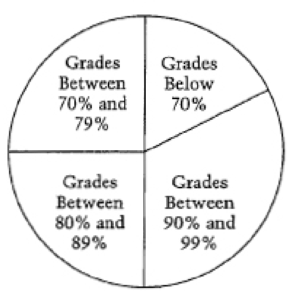
VB335159

Level I
Theta -1.42

Item VB335159.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z2M12.

MATHEMATICS GRADES



4. The circle graph above shows the distribution of grades for the 24 students in Shannon's mathematics class. Consider each of the following statements. Can the conclusion be made from the graph?

Fill in one oval to indicate YES or NO for each statement.

	Yes	No
(a) About $\frac{1}{2}$ of the class has a grade of 90% or better.	○	○
(b) Over $\frac{1}{2}$ of the class has a grade of 80% or better.	○	○
(c) There are no students with a grade of 60%.	○	○
(d) There are fewer students with a grade below 70% than there are between 70% and 79%.	○	○

HW000854

Level I
Theta 0.01

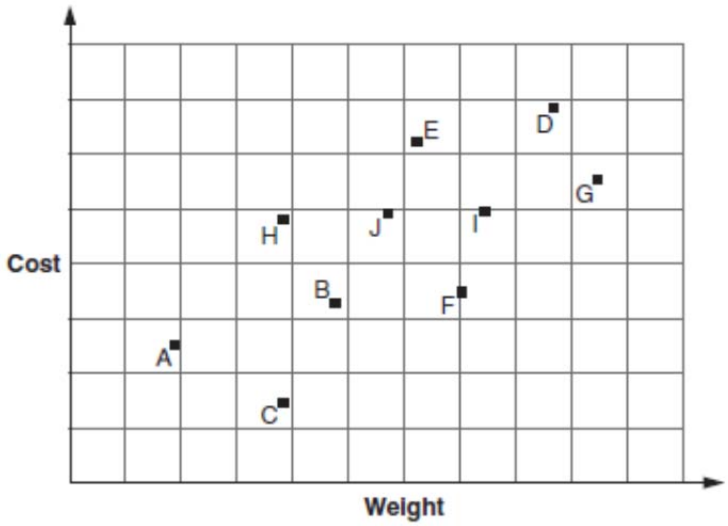
Item HW000854.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z23M8B.

Boxes of Candy

This problem gives you the chance to:
- interpret a scatter graph

This scatter graph shows the weights and the costs of 10 boxes of candy, A through J.



1. Which box of candy is the most expensive? ____________

2. Which two boxes of candy weigh the same? ____________

Level I
Theta 0.89

3. Which box of candy appears to be the best value for the money?

____

Explain how you found your answer.

4. What does the scatter graph show about the connection between the weights of the boxes of candy and how much they cost?

Level VII
Theta 1.21

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Level II

6. The results of a class survey on whether students liked a new television show are as follows.

25 students liked the new show.
15 students disliked the new show.
5 students had no opinion on the new show.

On the graph below, each represents 5 students. Draw the correct number of faces to illustrate the results of the class survey.


= 5 students

Liked	
Disliked	
No Opinion	

IY002250

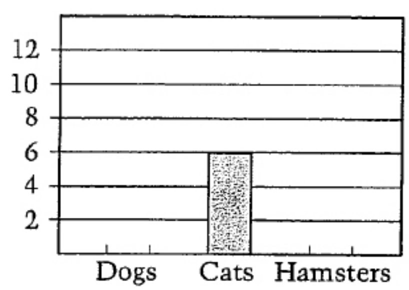
Item IY002250.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z12M3B.

7. Draw bars on the graph below so that the number of dogs is twice the number of cats and the number of hamsters is one-half the number of cats.

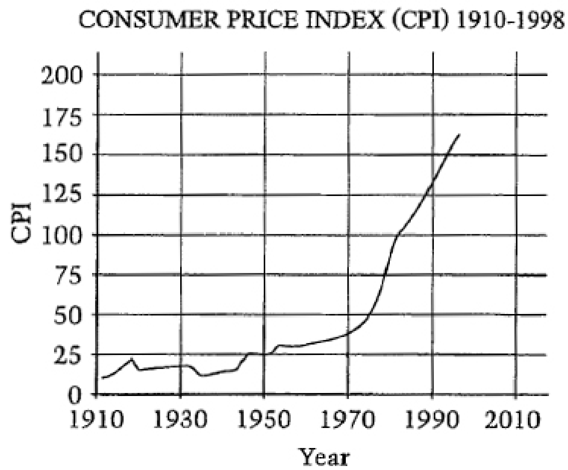
Level II
Theta -0.76

OM000557



Item OM000557.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z12M4B.



8. The 1990 Consumer Price Index (CPI) was <u>about</u> how many times the 1950 CPI ?

Level II
Theta 0.65

Ⓐ 2

Ⓑ 5

Ⓒ 10

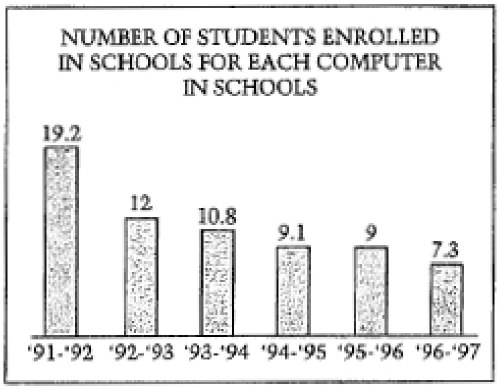
Ⓓ 25

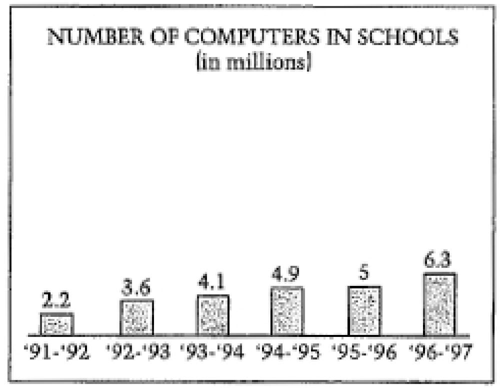
Ⓔ 100

YJ000102

Item YJ000102.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z23M8B**.**





13. Based on the information in the graphs above, how many students were enrolled in schools in '96-'97 ?

Level II
Theta 2.03

____

Show how you found your answer.

YJ000093

Item YJ000093.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z23M8B.

Level III

6. The prices of gasoline in a certain region are \$1.41, \$1.36, \$1.57, and \$1.45 per gallon. What is the median price per gallon for gasoline in this region?

Level III
Theta 0.99

Ⓐ \$1.41

Ⓑ \$1.43

Ⓒ \$1.44

Ⓓ \$1.45

Ⓔ \$1.47

VB335157

Item VB335157.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z2M12.

VB434825

11. For a school report, Luke contacted a car dealership to collect data on recent sales. He asked, "What color do buyers choose most often for their car?" White was the response. What statistical measure does the response "white" represent?

Level III
Theta 1.07

Ⓐ Mean

Ⓑ Median

Ⓒ Mode

Ⓓ Range

Ⓔ Interquartile range

Item VB434825.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block B2M10.

Level IV

Name	Age
Toni	60
Kim	59
Sue	59
Joe	56
Carlos	55
Lynn	52
Ray	51
Marta	20
Carl	10

18. The table above shows the ages of people at a picnic. Which of the following is the most appropriate statistic to use to best describe the "typical" age of the people at this picnic?

Level IV
Theta 1.1

Ⓐ Median

Ⓑ Mode

Ⓒ Mean

Ⓓ Range

Ⓔ Frequency

IY002422

Item IY002422.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z23M8B.

Ages

This problem gives you the chance to:
- show understanding of mean and range

1. Twelve people in an office have a mean age of 24 years, 0 months.
What do the ages of the 12 people add up to?

____

2. The oldest person in the office is 27 years, 8 months old.
The range of the ages of the people is 8 years, 10 months.
What is the age of the youngest person? ______________
Show your work.

3. A year later, the same 12 people are still working in the same office.
What is their mean age now?
Explain your answer. ______________

Level IV
Theta 1.39

4. What is the range of their ages now?
Explain your answer. ______________

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Note: Only Item 3 from Ages pertains to this progression. The remaining items 1, 2, 4, and 5 do not occur in this progression.

8. The table below shows the number of customers at Malcolm's Bike Shop for 5 days, as well as the mean (average) and the median number of customers for these 5 days.

Level IV
Theta 7.62

Number of Customers at Malcolm's Bike Shop	
Day 1	100
Day 2	87
Day 3	90
Day 4	10
Day 5	91
Mean (average)	75.6
Median	90

Which statistic, the mean or the median, best represents the typical number of customers at Malcolm's Bike Shop for these 5 days?

Explain your reasoning.

HL002246

Item HL002246.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z23M9B.

Level V

VB417888

2. Which of the following types of graph would be best to show the change in temperature recorded in a city every 15 minutes over a 24-hour period?

Level V
Theta -0.86

Ⓐ Pictograph

Ⓑ Circle graph

Ⓒ Line graph

Ⓓ Box-and-whisker plot

Ⓔ Stem-and-leaf plot

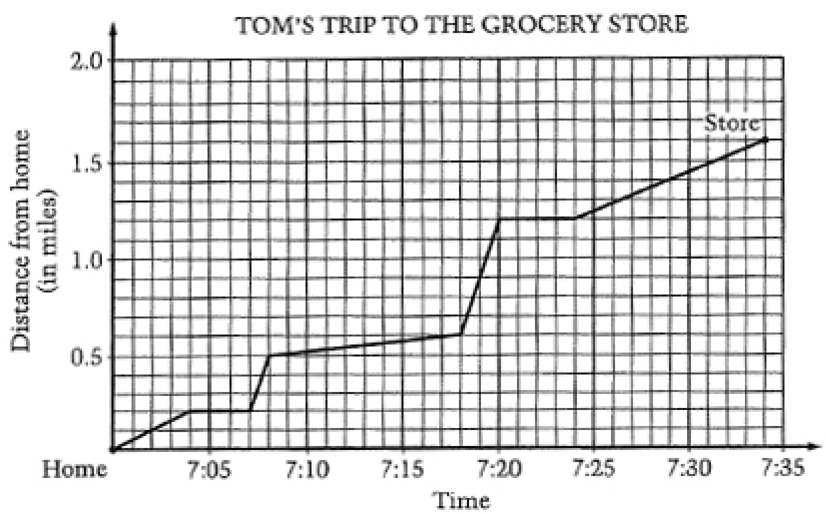
Item VB417888.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block B2M7**.**

VB434849

10. Tom went to the grocery store. The graph below shows Tom's distance from home during his trip.

Level V
Theta 1.18



Tom stopped twice to rest on his trip to the store. What is the total amount of time that he spent resting?

Ⓐ 5 minutes

Ⓑ 7 minutes

Ⓒ 8 minutes

Ⓓ 10 minutes

Ⓔ 25 minutes

Item VB434849.

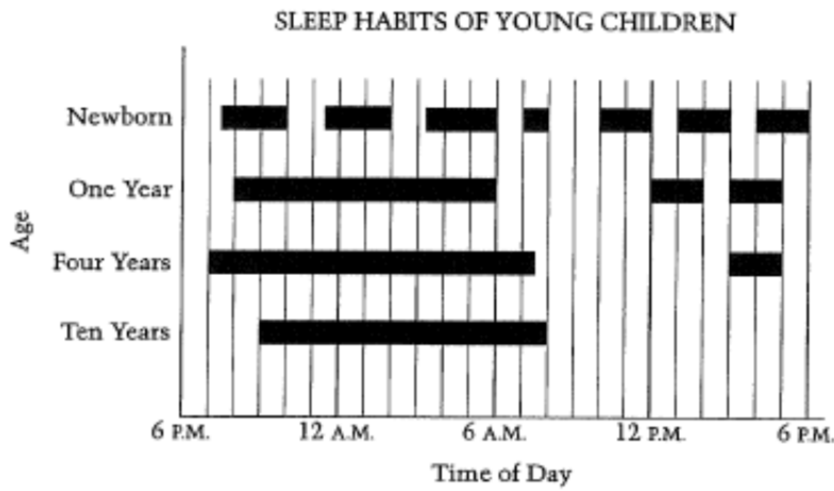
Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block B2M10**.**

Level V
Theta 1.56

18. The graph below and written summary on the next page present information about the sleep habits of newborn babies, one year olds, four year olds, and ten year olds. Each solid bar represents a period of sleep.

Some of the information presented in the summary does not agree with the information in the graph.

For example, there is an error in sentence 1 that has already been identified and corrected for you.



- In sentences 2 and 3 below, underline the information that is not correct based on the graph. There is an error in each sentence.
- Then, write the correct information above the errors in sentences 2 and 3.

(1) According to research that has been done on sleep habits and patterns of sleep in children, the number of hours that a newborn baby sleeps in a 24-hour period of time is *more* less than that of a ten year old.

(2) From the time a child is born until it reaches age ten, the number of different time periods of sleep increases as the child grows older.

(3) Newborns need 2 more hours of sleep than ten year olds between 6 a.m. and 6 p.m.

YJ000060

Item YJ000060.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z23M9B.

Level VI

Level VI
Theta 0.15

11. Benita and Jeff each surveyed some of the students in their eighth-grade homerooms to determine whether chicken or hamburgers should be served at the class picnic. The survey forms are shown below.

Benita's Survey		
Homeroom: 8-A		
Number of Students in Homeroom: 23		
Student Surveyed	Chicken	Hamburger
Adam	✓	
Carlene	✓	
Nancy	✓	
Hugh	✓	

Jeff's Survey		
Homeroom: 8-B		
Number of Students in Homeroom: 20		
Student Surveyed	Chicken	Hamburger
Becky		✓
Tanya	✓	
Joe	✓	
Ben		✓
Abby		✓
Lino	✓	
Marian		✓
Han		✓
Chris		✓
Tina		✓
Nate		✓
Darrell		✓

Benita reported that 100 percent of those in her survey wanted chicken.
Jeff reported that 75 percent of those in his survey wanted hamburger.

Which survey, Benita's or Jeff's, would probably be better to use when making the decision about what to serve?

Explain why that survey would be better.

AP000506

Item AP000506.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z23M9B.

Best Guess

This problem gives you the chance to:
- make and justify conclusions based on data
- compare sets of estimates and use mean and range

Aaron, Ben, and Claude want to see who can best estimate how long it takes for 30 seconds to go by. One person starts a stopwatch. One of the others tries to guess when 30 seconds have passed and then says "Stop." Each boy guesses five times. The timekeeper records the results.

Here are the results. All times are given in seconds.

Aaron's guesses	31	25	32	27	28
Ben's guesses	37	19	40	36	22
Claude's guesses	32	38	24	32	32

1. Who do you think is best at estimating how long it takes for 30 seconds to go by?

Level VI
Theta 3.97

____

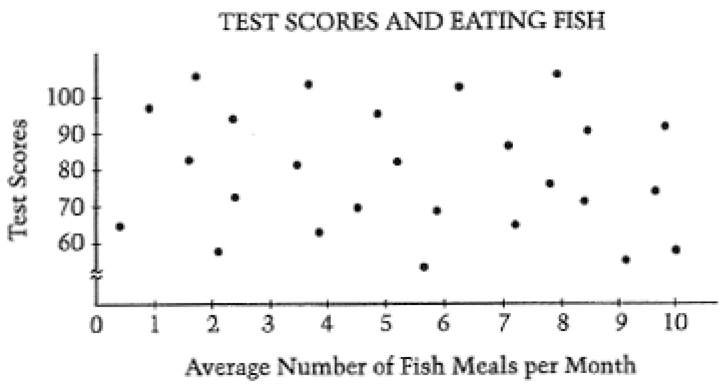
Show all your calculations.

2. Explain clearly the reasons for your choice.

Level VI
Theta 3.11

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Level VII



VB417891

Level VII
Theta 0.28

13. For a science project, Marsha made the scatterplot above that gives the test scores for the students in her math class and the corresponding average number of fish meals per month. According to the scatterplot, what is the relationship between test scores and the average number of fish meals per month?

Ⓐ There appears to be no relationship.

Ⓑ Students who eat fish more often score higher on tests.

Ⓒ Students who eat fish more often score lower on tests.

Ⓓ Students who eat fish 4-6 times per month score higher on tests than those who do not eat fish that often.

Ⓔ Students who eat fish 7 times per month score lower on tests than those who do not eat fish that often.

Item VB417891.

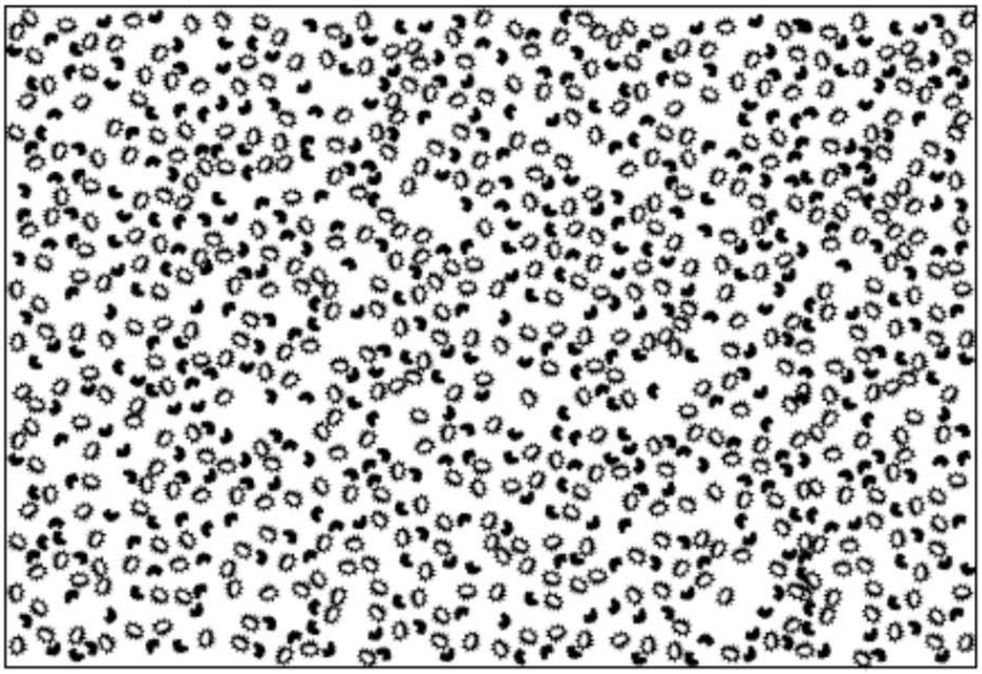
Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block B2M7.

Bacteria

This problem gives you the chance to:
- use a sampling strategy to estimate a large number

Two types of bacteria are shown on this microscope slide.

Some are **long**, with no "holes":  Some are **round**, with "holes": 



1. It would take a scientist a long time to count all these bacteria one by one. Describe a quicker method that could be used to **estimate** the **total** number of bacteria on the slide.

Level VII
Theta 0.67

2. Use your method to **estimate** the **total** number of bacteria on the slide.

Level VII
Theta 1.62

Level VII
Theta 2.26

3. Estimate the percentage of bacteria that are **round**.
Show your method clearly.

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

For Boxes of Candy Item 4 (Level VII, Theta 1.21), please see page 196.

Figure A-3. Equations Learning Progression Branch 1

Level I

$$\boxed{} - 8 = 21$$

6. What number should be put in the box to make the number sentence above true?

Answer: ____________________

HL000844

Level I
Theta -1.58

Item HL000844.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z12M5B.

Level II

VB417883

Level II
Theta -0.45

1. If $x = 2n + 1$, what is the value of x when $n = 10$?

Ⓐ 11

Ⓑ 13

Ⓒ 20

Ⓓ 21

Ⓔ 211

Item VB417883.

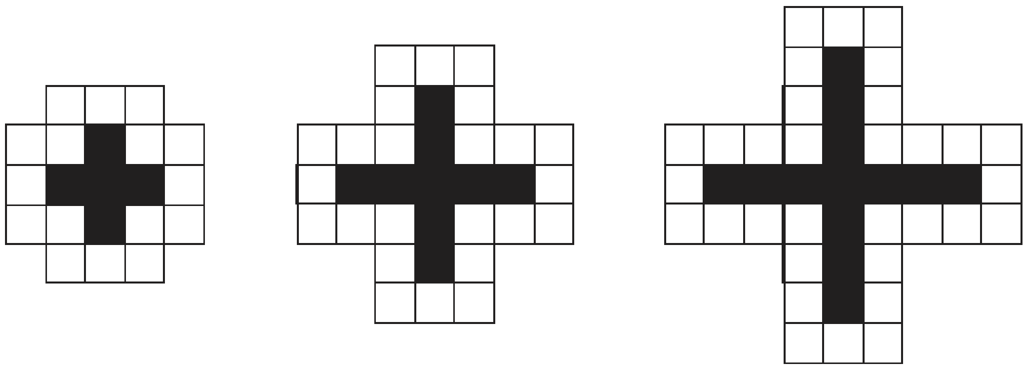
Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block B2M7.

Tiling Squares

This problem gives you the chance to:

- extend and check patterns
- derive formulas connecting different pairs of variables

Marcia is using black and white square tiles to make patterns.



Pattern 1 Pattern 2 Pattern 3

1. How many black tiles are needed to make Pattern 4? ______________ Level II Theta -0.29

Marcia begins to make a table to show the number of black and white tiles she is using.

Pattern number	1	2	3	4
Number of white tiles	16	24		
Number of black tiles	5	9		
Total	21	33		

2. Fill in the missing numbers in Marcia's table.

3. Marcia wants to know how many white tiles and black tiles there will be in the tenth pattern, but she does not want to draw all the patterns and count the squares.
Explain or show another way she could find her answer.

____

____

____

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Note: Only Item 1 from Tiling Squares pertains to this progression. The remaining items 2 and 3, as well as the omitted items 4, 5, and 6, do not occur in this progression.

6. If m represents the total number of months that Jill worked and p represents Jill's average monthly pay, which of the following expressions represents Jill's total pay for the months she worked?

Level II
Theta -0.14

Ⓐ $m + p$

Ⓑ $m \div p$

Ⓒ $m \times p$

Ⓓ $p \div m$

Ⓔ $m - p$

VB434929

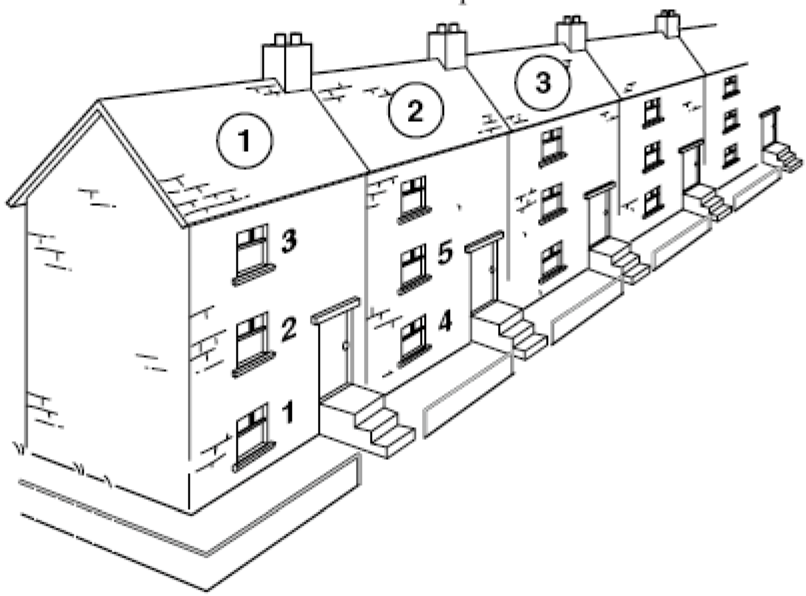
Item VB434929.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block B2M7.

Apartment Numbers

This problem gives you the chance to:
- see and work with number patterns
- express number patterns in words and explain an error

A long row of houses has been changed into apartments.
Each house has been made into three apartments.



The apartments are numbered in order: basement, middle, and top, for each house in the row. Apartments numbered 1 to 5 are shown in the drawing.

1. Complete the following table to show the apartment numbers for the first five houses.

Level II
Theta -0.05

House	Basement	Middle apartment	Top apartment
1	1	2	3
2	4	5	
3			
4			
5			

2. Mrs. Smith lives in the top apartment in the tenth house.
What is the number of her apartment? ______________

3. Mr. Patel and Mr. Dobson are next door neighbors.
They both have basement apartments. Mr. Dobson lives in apartment 25.
What are the possible numbers of Mr. Patel's apartment? ______________

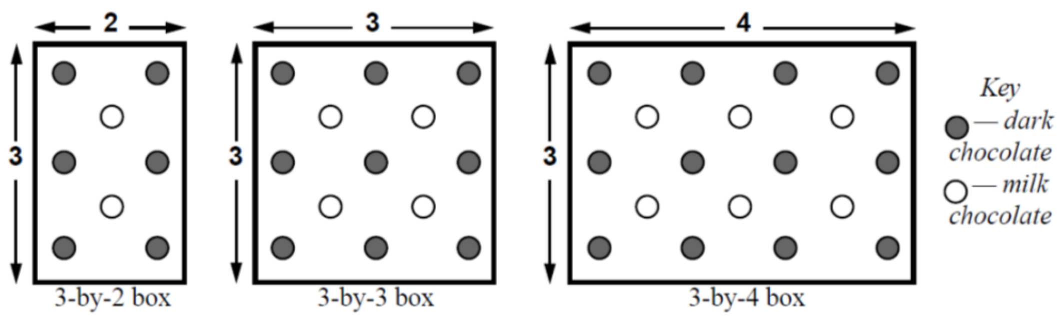
Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Note: Only Item 1 from Apartment Numbers pertains to this progression. The remaining items 2 and 3, as well as the omitted items 4 and 5, do not occur in this progression.

Boxes of Chocolates

This problem gives you the chance to:
- find and extend a number pattern
- express the pattern using a rule or formula

Sam designs and makes boxes for chocolate candies.
The boxes have different lengths, but they are all the same width.
The chocolates are always arranged in the same kind of pattern.
The shaded circles show dark chocolates.
The white circles show milk chocolates.



Sam makes a table to show how many chocolates are in each size of box.

Size of box	3×2	3×3	3×4	3×5	3×6
Number of dark chocolates	6	9			
Number of milk chocolates	2	4			
Total number of chocolates	8	13			

1. Fill in the missing numbers in Sam's table.

2. Describe two number patterns you can see in the table.

Level II
Theta 0.42

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Note: Only Item 1 from Boxes of Chocolates pertains to this progression. The remaining item 2 as well as the omitted items 3, 4, and 5 do not occur in this progression.

2. Consider each of the following expressions. In each case, does the expressior equal $2x$ for all values of x ?

Fill in one oval to indicate YES or NO for each expression.

Level II
Theta 1.69

		Yes	No
(a)	2 times x	○	○
(b)	x plus x	○	○
(c)	x times x	○	○

EL

Item EL001490.

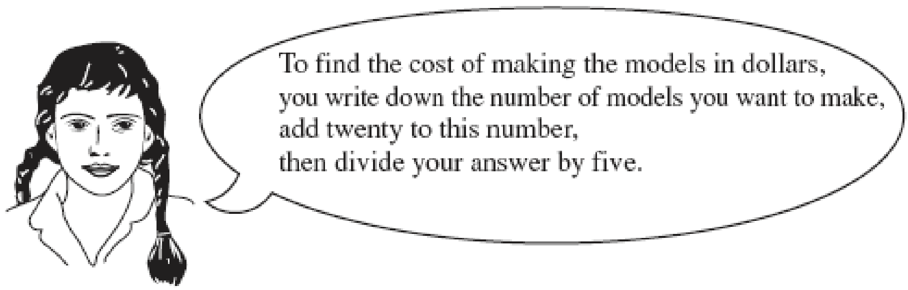
Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z23M9B.

Level III

Emma's Models

This problem gives you the chance to:
- use tables, graphs, and formulas to solve problems

Emma is making some clay models to sell at the school fair.

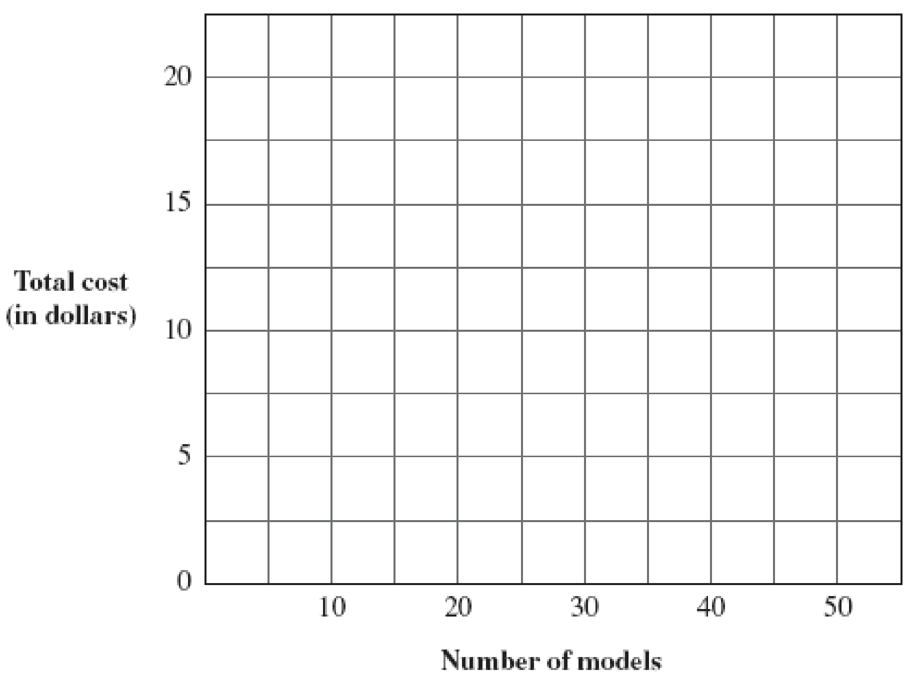


1. Complete the table below to show how the cost depends on the number of models Emma makes. The first value has been calculated and written in the table.

Number of models	10	20	30	40	50
Total cost (in dollars)	6				

Level III
Theta -0.33

2. Draw a graph that shows the information in the table above.



Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Note: Only Item 1 from Emma's Models pertains to this progression. The remaining item 2, as well as the omitted items 3 and 4, do not occur in this progression.

Party

This problem gives you the chance to:
- choose and use number operations in context
- find and use an algebraic formula
- relate formulae and graphs

Sarah is organizing a party at the Vine House Hotel.

1. Sarah thinks there will be 60 people at the party.
Show that the cost will be $1350.

Level III
Theta 0.31

2. What is the cost of a party for 100 people at the Vine House Hotel? $ ____________
Show how you figured it out.

3. C dollars is the cost of a party for P people.
Find a formula that gives C in terms of P.

____

____

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Note: Only Item 1 from Party pertains to this progression. The remaining items 2 and 3, as well as the omitted items 4 and 5, do not occur in this progression.

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
Number Sold, n	4	0	5	2	3	6
Profit, p	\$2.00	\$0.00	\$2.50	\$1.00	\$1.50	\$3.00

Level III
Theta 0.60

VB335172

15. Angela makes and sells special-occasion greeting cards. The table above shows the relationship between the number of cards sold and her profit. Based on the data in the table, which of the following equations shows how the number of cards sold and profit (in dollars) are related?

Ⓐ $p = 2n$

Ⓑ $p = 0.5n$

Ⓒ $p = n - 2$

Ⓓ $p = 6 - n$

Ⓔ $p = n + 1$

Item VB335172.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block B2M7**.**

VB434848

8. The length of a rectangle is 3 feet less than twice the width, w (in feet). What is the length of the rectangle in terms of w?

Ⓐ $3 - 2w$

Ⓑ $2(w + 3)$

Ⓒ $2(w - 3)$

Ⓓ $2w + 3$

Ⓔ $2w - 3$

Level III
Theta 0.68

VB434848.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block B2M10**.**

x	y
0	−1
1	2
2	5
3	8
10	29

17. Which of the following equations represents the relationship between x and y shown in the table above?

Ⓐ $y = x^2 + 1$

Ⓑ $y = x + 1$

Ⓒ $y = 3x - 1$

Ⓓ $y = x^2 - 3$

Ⓔ $y = 3x^2 - 1$

Level III
Theta 0.71

VB335163

Item VB335163.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z2M12.

11. If the grid in Question 10 were large enough and the beetle continued to move in the same pattern, would the point that is 75 blocks up and 100 blocks over from the starting point be on the beetle's path?

Level III
Theta 2.3

Ⓐ Yes Ⓑ No

Give a reason for your answer.

____

____

____

XH000443

Item XH000443.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z12M4B.

Note: See Item XH000442 on page 179 (first item in graphing learning progression) for the graph referenced in this question.

Level IV

3. Which of the following is equal to $6(x + 6)$?

Ⓐ $x + 12$

Ⓑ $6x + 6$

Ⓒ $6x + 12$

Ⓓ $6x + 36$

Ⓔ $6x + 66$

Level IV
Theta 1.02

Item VB335154.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z2M12.

Level V

4. If $15 + 3x = 42$, then $x =$

Ⓐ 9

Ⓑ 11

Ⓒ 12

Ⓓ 14

Ⓔ 19

Level V
Theta -0.53

Item YJ000107.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z23M9B**.**

VB335169

1. Which of the following equations has the same solution as the equation $2x + 6 = 32$?

Level V
Theta 0.48

Ⓐ $2x = 38$

Ⓑ $x - 3 = 16$

Ⓒ $x + 6 = 16$

Ⓓ $2(x - 3) = 16$

Ⓔ $2(x + 3) = 32$

Item VB335169.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block B2M10.

16. The w in the inequality $8w - 4 > 5$ is replaced by each of the numbers 0, 1, 2, and 3. For which of these numbers is the inequality true?

Level V
Theta 0.69

Ⓐ 0

Ⓑ 1

Ⓒ 2, 3

Ⓓ 1, 2, 3

Ⓔ None of the numbers

AP00071

Item AP000710.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z23M8B.

Level VI

VB434852

9. The formula $d = 16t^2$ gives the distance d, in feet, that an object has fallen t seconds after it is dropped from a bridge. A rock was dropped from the bridge and its fall to the water took 4 seconds. According to the formula, what is the distance from the bridge to the water ?

Ⓐ 16 feet

Ⓑ 64 feet

Ⓒ 128 feet

Ⓓ 256 feet

Ⓔ 4,096 feet

Level VI
Theta 0.93

Item VB434852.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block B2M7.

10. In the equation $y = 4x$, if the value of x is increased by 2, what is the e on the value of y ?

Ⓐ It is 8 more than the original amount.

Ⓑ It is 6 more than the original amount.

Ⓒ It is 2 more than the original amount.

Ⓓ It is 16 times the original amount.

Ⓔ It is 8 times the original amount.

Level VI
Theta 1.71

Item HW000857.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z12M3B.

Figure A-4. Equations Learning Progression Branch 2

Level I

$$\boxed{} - 8 = 21$$

6. What number should be put in the box to make the number sentence above true?

Answer: ____________________

HL000844

Level I
Theta -1.58

Item HL000844.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z12M5B.

Level II

VB417883

1. If $x = 2n + 1$, what is the value of x when $n = 10$?

Ⓐ 11

Ⓑ 13

Ⓒ 20

Ⓓ 21

Ⓔ 211

Level II
Theta -0.45

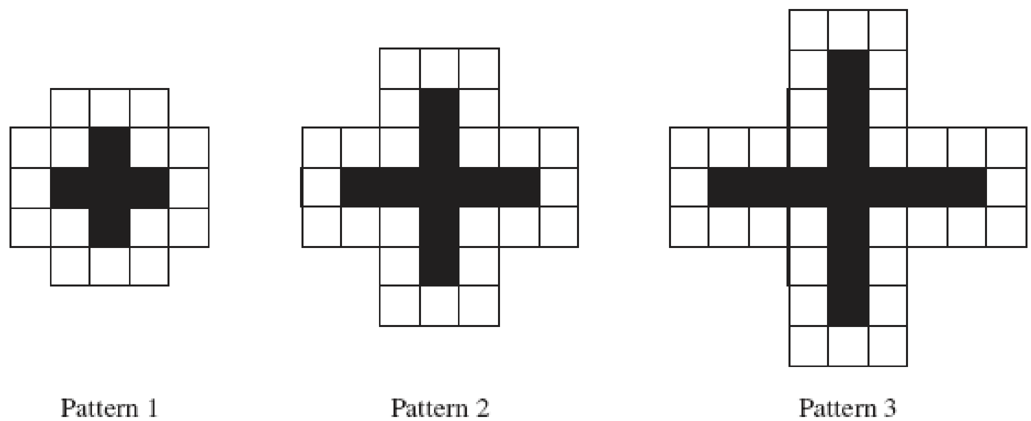
Item VB417883.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block B2M7.

Tiling Squares

This problem gives you the chance to:
- extend and check patterns
- derive formulas connecting different pairs of variables

Marcia is using black and white square tiles to make patterns.



1. How many black tiles are needed to make Pattern 4? ____________

Level II
Theta -0.29

Marcia begins to make a table to show the number of black and white tiles she is using.

Pattern number	1	2	3	4
Number of white tiles	16	24		
Number of black tiles	5	9		
Total	21	33		

2. Fill in the missing numbers in Marcia's table.

Level IV
Theta 1.27

3. Marcia wants to know how many white tiles and black tiles there will be in the tenth pattern, but she does not want to draw all the patterns and count the squares.
Explain or show another way she could find her answer.

Level VII
Theta 1.56

4. Using W for the number of white tiles and P for the pattern number, write down a rule or formula linking W with P.

Level VII
Theta 1.5

5. Using B for the number of black tiles and P for the pattern number, write down a rule or formula linking B with P.

Level VII
Theta 1.62

6. Now, using T for the total number of tiles and P for the pattern number, write down a rule or formula linking T with P.

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

6. If m represents the total number of months that Jill worked and p represents Jill's average monthly pay, which of the following expressions represents Jill's total pay for the months she worked?

Level II
Theta -0.14

Ⓐ $m + p$

Ⓑ $m \div p$

Ⓒ $m \times p$

Ⓓ $p \div m$

Ⓔ $m - p$

VB434929

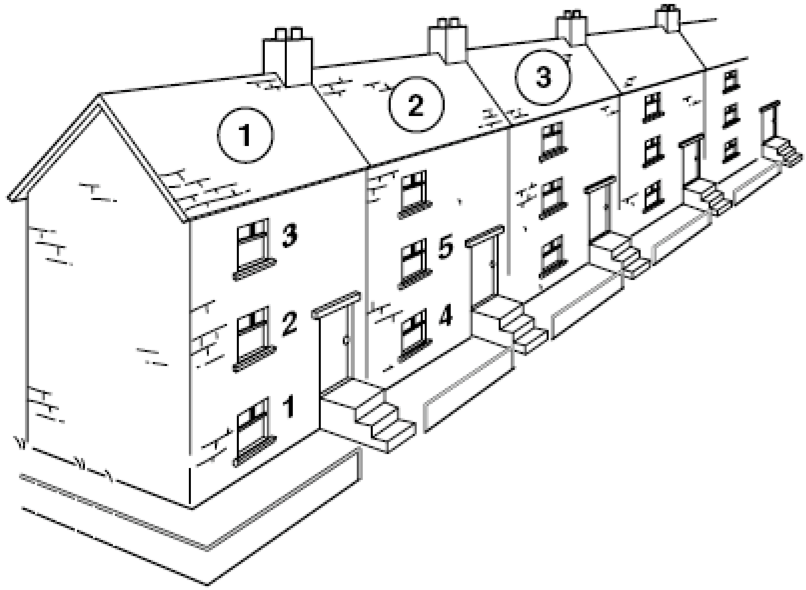
Item VB434929.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z2M11.

Apartment Numbers

This problem gives you the chance to:
- see and work with number patterns
- express number patterns in words and explain an error

A long row of houses has been changed into apartments.
Each house has been made into three apartments.



The apartments are numbered in order: basement, middle, and top, for each house in the row. Apartments numbered 1 to 5 are shown in the drawing.

1. Complete the following table to show the apartment numbers for the first five houses.

Level II
Theta -0.05

House	Basement	Middle apartment	Top apartment
1	1	2	3
2	4	5	
3			
4			
5			

2. Mrs. Smith lives in the top apartment in the tenth house.
What is the number of her apartment? ______________

Level IV
Theta -0.49

3. Mr. Patel and Mr. Dobson are next door neighbors.
They both have basement apartments. Mr. Dobson lives in apartment 25.
What are the possible numbers of Mr. Patel's apartment? ______________

Level VI
Theta 1.10

Page 6 Apartment Numbers Book 4

4. Ms. Sanchez uses a rule to make a table that shows some of the house numb the numbers of the middle apartments.

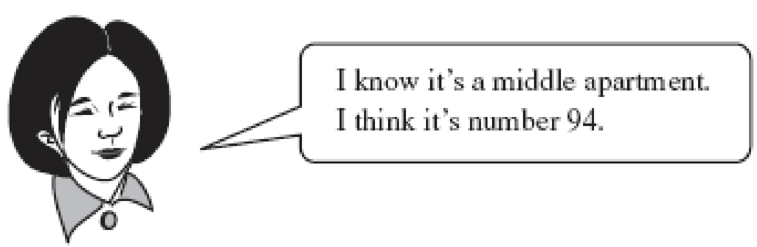
Level VII
Theta 1.39

House number	Middle apartment number
2	5
3	8
6	17
10	29
12	35
25	74

Write down what you think Ms. Sanchez's rule is.

5. Miss Ling is going to visit her friend.

Level VII
Theta 1.52



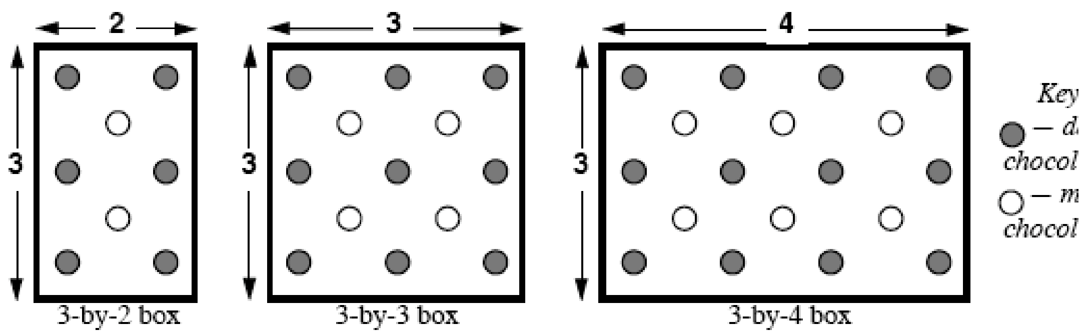
Is Miss Ling correct? Explain your answer.

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Boxes of Chocolates

This problem gives you the chance to:
- find and extend a number pattern
- express the pattern using a rule or formula

Sam designs and makes boxes for chocolate candies.
The boxes have different lengths, but they are all the same width.
The chocolates are always arranged in the same kind of pattern.
The shaded circles show dark chocolates.
The white circles show milk chocolates.



Sam makes a table to show how many chocolates are in each size of box.

Size of box	3×2	3×3	3×4	3×5	3×6
Number of dark chocolates	6	9			
Number of milk chocolates	2	4			
Total number of chocolates	8	13			

Level II
Theta 0.42

1. Fill in the missing numbers in Sam's table.

2. Describe two number patterns you can see in the table.

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Note: Only Item 1 from Boxes of Chocolates pertains to this progression. The remaining item 2, as well as the omitted items 3, 4, and 5, do not occur in this progression.

2. Consider each of the following expressions. In each case, does the expression equal $2x$ for all values of x ?

Fill in one oval to indicate YES or NO for each expression.

Level II
Theta 1.69

		Yes	No
(a)	2 times x	○	○
(b)	x plus x	○	○
(c)	x times x	○	○

EL001490

Item EL001490.

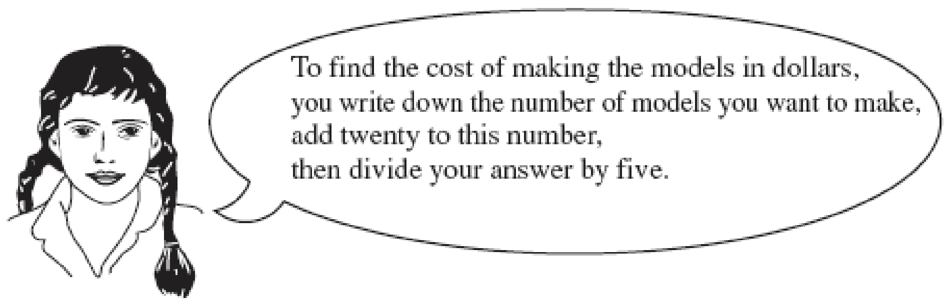
Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z23M9B.

Level III

Emma's Models

This problem gives you the chance to:
- use tables, graphs, and formulas to solve problems

Emma is making some clay models to sell at the school fair.

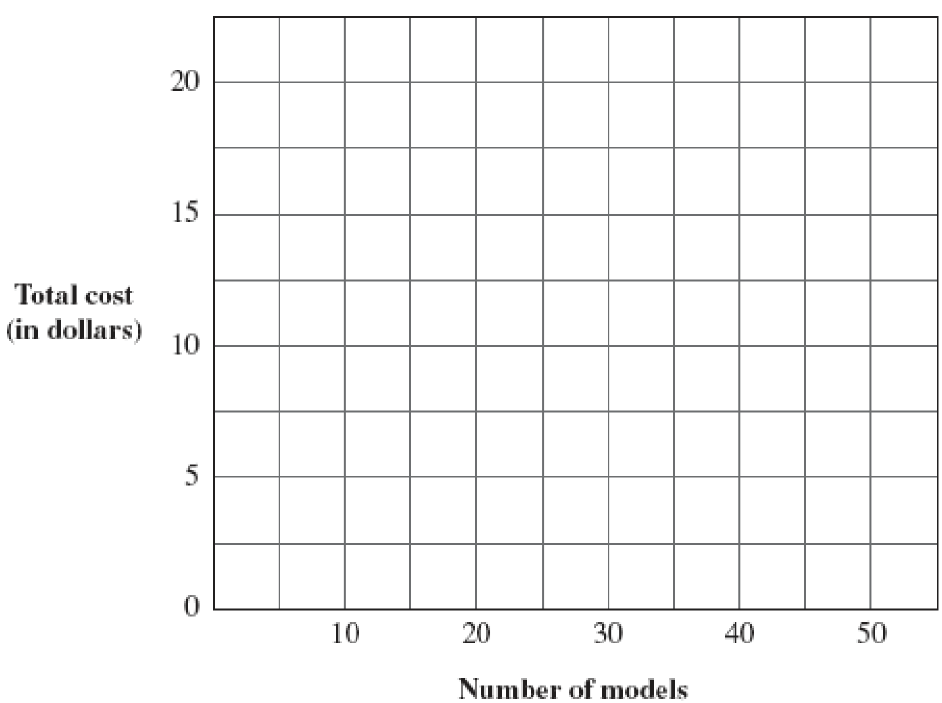


1. Complete the table below to show how the cost depends on the number of models Emma makes. The first value has been calculated and written in the table.

Number of models	10	20	30	40	50
Total cost (in dollars)	6				

Level III
Theta -0.33

2. Draw a graph that shows the information in the table above.



3. Write an algebraic expression that shows how much it costs Emma in dollars to make n models.

____

4. Emma spends \$30 making her models. How many models does she make? Show your work.

Level VII
Theta 0.63

____

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Note: Only Items 1 and 4 from Emma's Models pertain to this progression. Items 2 and 3 do not occur in this progression.

Party

This problem gives you the chance to:

- choose and use number operations in context
- find and use an algebraic formula
- relate formulae and graphs

Sarah is organizing a party at the Vine House Hotel.

1. Sarah thinks there will be 60 people at the party. Level III
Show that the cost will be $1350. Theta 0.31

2. What is the cost of a party for 100 people at the Vine House Hotel? $ __________ Level IV
Show how you figured it out. Theta 0.73

3. C dollars is the cost of a party for P people.
Find a formula that gives C in terms of P. Level VII
Theta 1.98

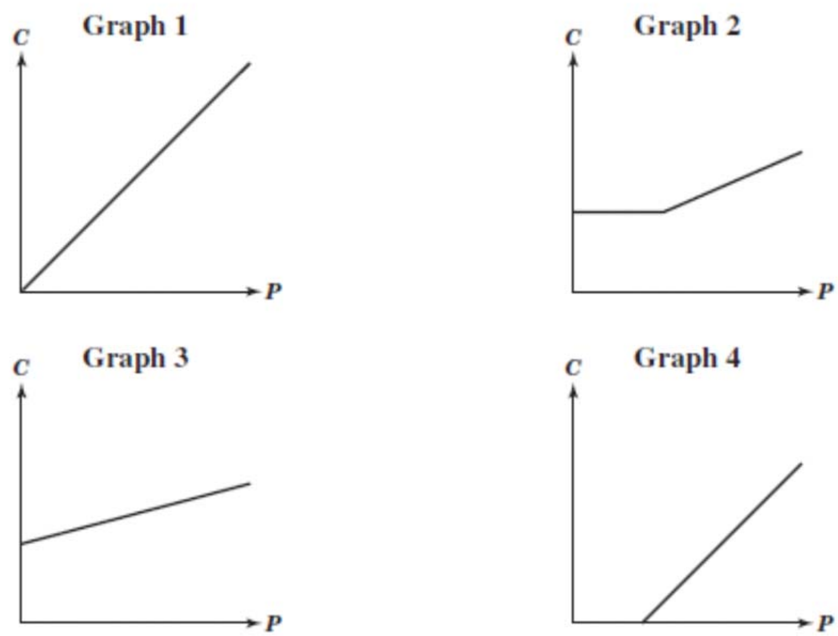
____

____

4. Sarah's party cost \$1750 in all.
How many people came to the party?
Show your calculations.

Level VI
Theta 1.20

5. Which of these graphs shows the connection between the number of people at the party, *P*, and the cost, *C*?



Explain how you figured it out.

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Note: Only Items 1 through 4 from Party pertain to this progression. Item 5 does not occur in this progression.

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
Number Sold, n	4	0	5	2	3	6
Profit, p	\$2.00	\$0.00	\$2.50	\$1.00	\$1.50	\$3.00

VB335172

15. Angela makes and sells special-occasion greeting cards. The table above shows the relationship between the number of cards sold and her profit. Based on the data in the table, which of the following equations shows how the number of cards sold and profit (in dollars) are related?

Level III
Theta 0.60

Ⓐ $p = 2n$

Ⓑ $p = 0.5n$

Ⓒ $p = n - 2$

Ⓓ $p = 6 - n$

Ⓔ $p = n + 1$

Item VB335172.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block B2M7**.**

VB434848

8. The length of a rectangle is 3 feet less than twice the width, w (in feet). What is the length of the rectangle in terms of w?

Level III
Theta 0.68

Ⓐ $3 - 2w$

Ⓑ $2(w + 3)$

Ⓒ $2(w - 3)$

Ⓓ $2w + 3$

Ⓔ $2w - 3$

Item VB434848.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block B2M10**.**

x	y
0	−1
1	2
2	5
3	8
10	29

17. Which of the following equations represents the relationship between x and y shown in the table above?

Ⓐ $y = x^2 + 1$

Ⓑ $y = x + 1$

Ⓒ $y = 3x - 1$

Ⓓ $y = x^2 - 3$

Ⓔ $y = 3x^2 - 1$

Level III
Theta 0.71

VB335163

Item VB335163.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z2M12**.**

11. If the grid in Question 10 were large enough and the beetle continued to move in the same pattern, would the point that is 75 blocks up and 100 blocks over from the starting point be on the beetle's path?

Ⓐ Yes Ⓑ No

Give a reason for your answer.

Level III
Theta 2.3

XH000443

Item XH000443.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z12M4B.

Note: See Item XH000442 on page 179 (first item in graphing learning progression) for the graph referenced in this question.

Level IV

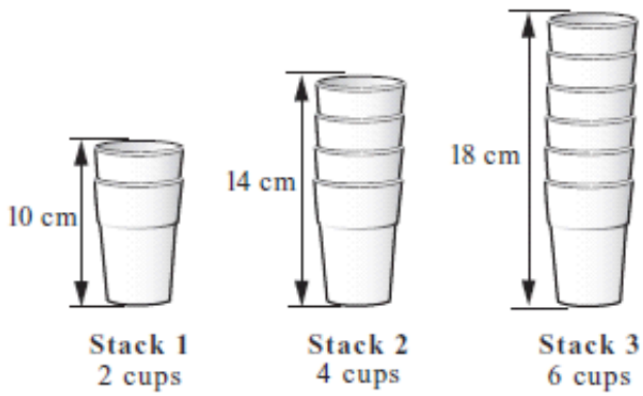
For Apartment Item 2 (Level IV, Theta -0.49), please see page 230.

Cups

This problem gives you the chance to:

- extend and work with a given pattern
- find and express the rule

Tom is stacking white plastic cups.
He measures the height of each stack.



Tom makes a table to show the number of white cups in each stack and the height of each stack.

Number of white cups	2	4	6	8
Height of stack of white cups in cm	10	14		

1. Fill in the missing numbers in Tom's table.

Level IV
Theta 1.17

2. Find the height of a stack of 12 white plastic cups. Explain how you figured it out.

____

____

3. Use Tom's table to figure out the height of 1 cup. Explain how you figured it out.

Level IV
Theta 0.96

____

____

Tom also stacks some brown plastic cups. He makes a table to show different numbers of brown cups and the height of each stack.

Number of brown cups	2	3	4	5	6	7
Height of stack of brown cups in cm	10				22	25

4. Fill in the missing numbers in Tom's table.

5. Use Tom's table to figure out the height of 1 brown cup. Show how you did it.

Level IV
Theta 0.32

____

____

6. Find a rule to calculate the height of a stack of any number of brown cups.

Level VII
Theta 1.65

____

____

7. A stack of 2 white plastic cups is 10 centimeters high.
A stack of 2 brown plastic cups is also 10 centimeters high.
Explain why a stack of 10 brown plastic cups is taller than a stack of 10 white plastic cups.

Level VIII
Theta 2.07

____

____

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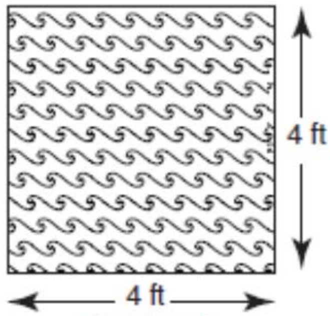
Fish Ponds

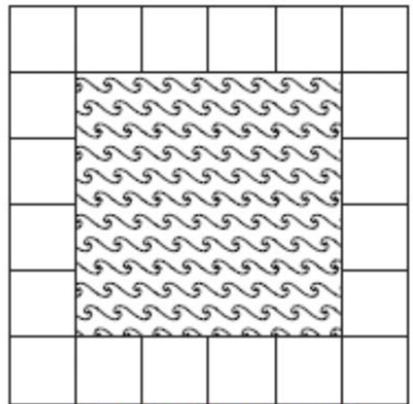
This problem gives you the chance to:

- find a number pattern in real spatial context and express the rule
- extend the rule to two variables

.



Fish Pond



Fish Pond surrounded by paving stones

Chris works at a garden center that sells square fish ponds and paving stones.

The paving stones are squares with sides one foot long.

1. Use the diagram above to figure out how many paving stones are needed to surround a fish pond that is 4 feet by 4 feet. __________

Level IV
Theta 0.59

2. Chris begins to make a table to show how many paving stones are needed to surround square ponds of different sizes. Fill in the empty boxes in the table.

Side of pond in feet	1	2	3	4	5
Number of paving stones	8				

3. How many paving stones are needed to surround a fish pond that is 20 feet by 20 feet? Explain how you figured it out.

Level V
Theta 0.61

4. Chris has 48 paving stones. Find the size of the largest square pond the paving stones can surround. Explain how you figured it out.

Level VI
Theta 0.79

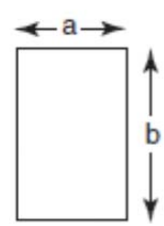
5. The garden center sells many different sizes of square fish ponds.

Write down a rule that will help Chris figure out how many paving stones are needed to surround square ponds of different sizes.

Level VII
Theta 0.87

6. The garden center decides to sell rectangular ponds.

Find a rule that will help Chris figure out how many paving stones are needed to surround rectangular ponds of different sizes.



Level VII
Theta 1.22

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

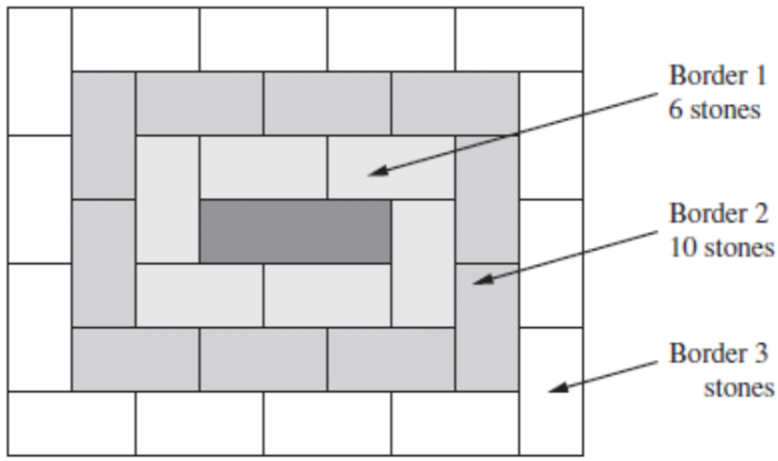
For Party Item 2 (Level IV, Theta 0.73), please see page 236.

Design a Garden

This problem gives you the chance to:

- find and extend a number pattern
- find the rule of the pattern and express the rule in words or algebra

The diagram below is a scale drawing showing Dave's garden plan.



The rectangle across the center is for planting roses. It measures 3 feet by 1 foot.

The borders will be made using colored rectangular stones. The stones measure 2 feet by 1 foot.

Dave decides to use this plan for three more borders.

He begins to make a table to find out how many stones he needs for each border.

Border	1	2	3	4	5
Number of stones	6	10			

1. Fill in the missing numbers in Dave's table. Explain how you figured out your answer.

2. Find a rule or formula for figuring out how many stones Dave needs for Border *n*.
Explain your reasoning.

3. Use your rule or formula to find the number of stones needed for Border 11.
Show your work.

Level IV
Theta 0.91

4. Dave has **96 stones.**
How many borders in all can he make, beginning with Border 1?
Explain how you figured it out.

Level VI
Theta 2.16

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

For Cups Item 3 (Level IV, Theta 0.96), please see page 240.

For Cups Item 2 (Level IV, Theta 1.17), please see page 240.

For Tiling Squares Item 2 (Level IV, Theta 1.27), please see page 227.

Level V

For Fish Ponds Item 3 (Level V, Theta 0.61), please see page 243.

Vacations

This problem gives you the chance to:
- analyze relationships using graphs and algebra

Here is some information about how some students are paying for their summer vacations.

Carla: Her mom gave her $100 in January and Carla has saved $25 every month since, starting in February.

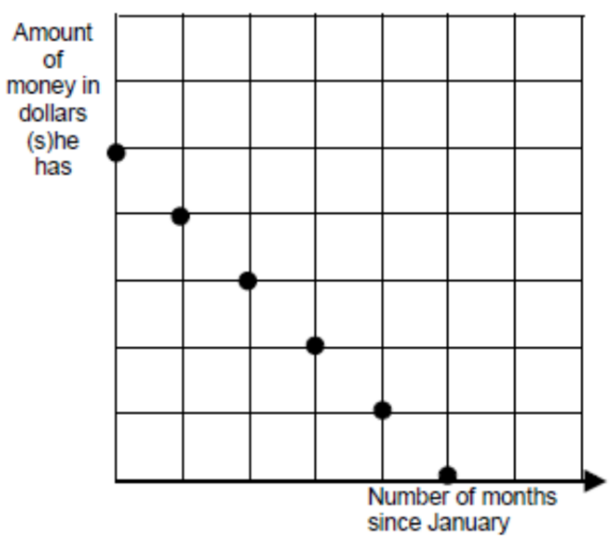
Arnie: Arnie put $150 in his piggy bank in January.

Sue: Sue booked her vacation in January. She had $250 in her piggy bank. Starting in February, she is paying $50 each month to the travel company.

Ben: Starting in February, Ben saves $30 every month.

Here are some graphs illustrating these situations.

1. Match each person with a graph and explain how you decided.



Name: ____________________

Reason: ____________________________

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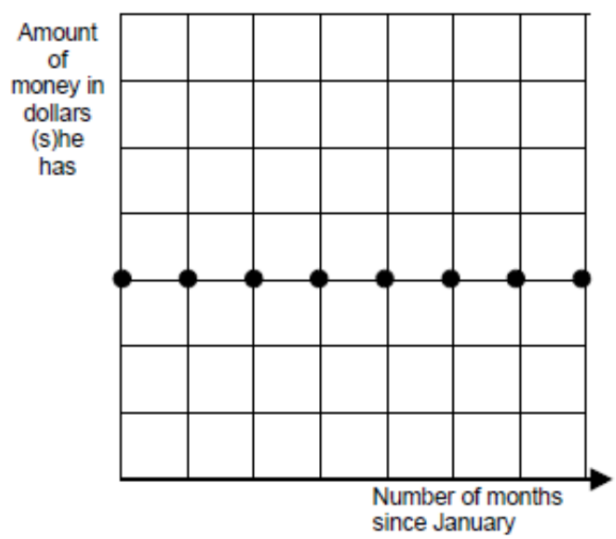

Name: ____________________

Reason: ____________________________

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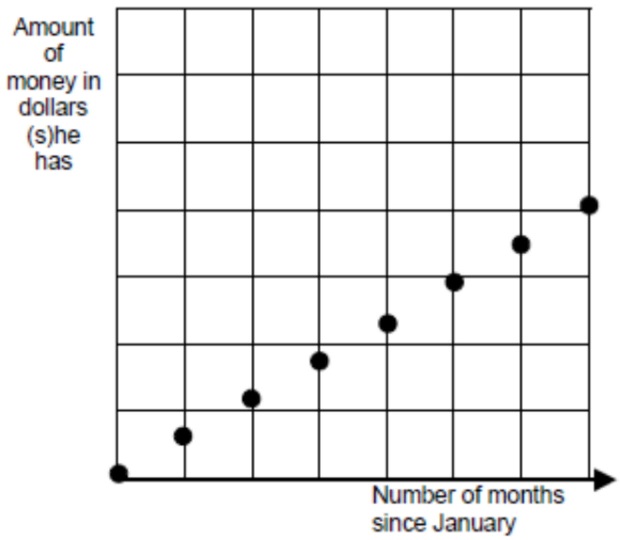

Name: ____________________

Reason: ____________________________

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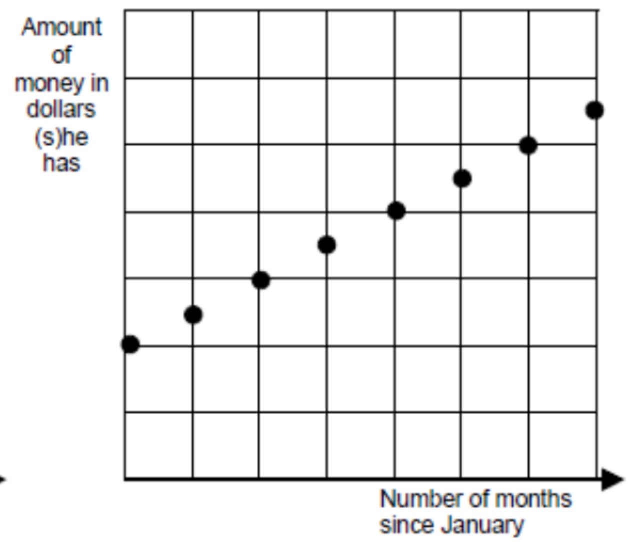

Name: ____________________

Reason: ____________________________

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2. In these equations, $A is the amount of money and n is the number of months since January.

$A = 250 - 50n$
$A = 30n$
$A = 150$

a. Find the person for each of these equations.
b. Write a formula for the fourth person.

Carla ______________________________

Arnie ______________________________

Sue ________________________________

Ben ________________________________

3. Write a possible description for this formula: $A = 50n + 150$

Level V
Theta 0.66

____

____

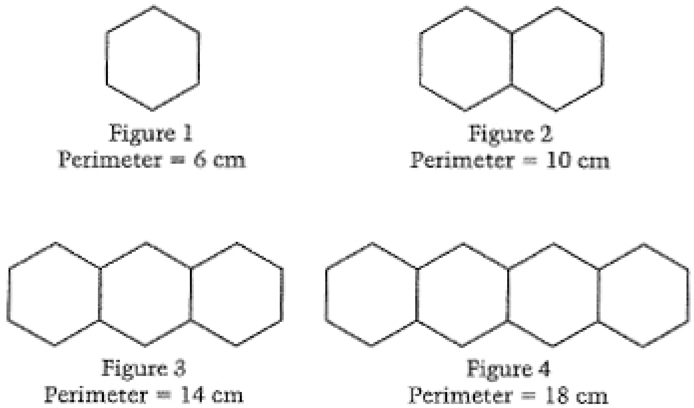
Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Note: Only Item 3 from Vacations pertains to this progression. The remaining items 1 and 2 do not occur in this progression.

VB434859

Level V
Theta 2.05

14. Each figure in the pattern below is made of hexagons that measure 1 centimeter on each side.



If the pattern of adding one hexagon to each figure is continued, what will be the perimeter of the 25th figure in the pattern?

Show how you found your answer.

Item VB434859.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block B2M7.

Level VI

For Fish Ponds Item 4 (Level VI, Theta 0.79), please see page 243.

For Apartment Numbers Item 3 (Level VI, Theta 1.10), please see page 230.

For Party Item 4 (Level VI, Theta 1.20), please see page 237.

For Design a Garden Item 4 (Level VI, Theta 2.16), please see page 245.

Level VII

For Emma's Models Item 4 (Level VII, Theta 0.63), please see page 235.

For Fish Ponds Item 5 (Level VII, Theta 0.87), please see page 243.

10. Sarah has a part-time job at Better Burgers restaurant and is paid \$5.50 for each hour she works. She has made the chart below to reflect her earnings but needs your help to complete it.

Level VII
Theta 1.20

EL001486

(a) Fill in the missing entries in the chart.

Hours Worked	Money Earned (in dollars)
1	\$5.50
4	
	\$38.50
$7\frac{3}{4}$	\$42.63

(b) If Sarah works h hours, then, in terms of h, how much will she earn?

Item EL001486.

Source: U.S. Department of Education, National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment, Grade 8, Block Z23M9B.

For Fish Ponds Item 6 (Level VII, Theta 1.22), please see page 243.

For Apartment Numbers Item 4 (Level VII, Theta 1.39), please see page 231.

For Tiling Squares Item 4 (Level VII, Theta 1.50), please see page 228.

For Apartment Numbers Item 5 (Level VII, Theta 1.52), please see page 231.

For Tiling Squares Item 3 (Level VII, Theta 1.56), please see page 227.

For Tiling Squares Item 5 (Level VII, Theta 1.62), please see page 228.

For Cups Item 6 (Level VII, Theta 1.65), please see page 241.

For Party Item 3 (Level VII, Theta 1.98), please see page 236.

Level VII

For Cups Item 7 (Level VIII, Theta 2.07), please see page 241.

Picking Apples

This problem gives you the chance to:
- work out costs from given rules

Anna goes to pick apples. She sees two orchards next to each other: David's orchard and Pam's orchard.
The signs below are at the entrance to the orchards.

DAVID'S APPLE ORCHARD Pick your own apples!	PAM'S ORCHARD DELICIOUS APPLES
	$10 entry fee
First 10 pounds $2 per pound	First 10 pounds $1.50 per pound
Each additional pound $1 per pound	Each additional pound $0.75

Anna wants to pick 40 pounds of apples.

1. a. How much does this cost at David's orchard? ________________________

Show your calculations.

b. How much does it cost at Pam's orchard? ________________________

Show your calculations.

2. Chris has $30 to spend.

   a. How many pounds of apples will he get if he goes to David's orchard? ________________
   Explain how you figured it out.

   ____

   ____

   ____

   b. If Chris goes to Pam's orchard, how many pounds of apples will he get? ________________
   Explain how you figured it out.

   ____

   ____

   ____

Level VIII
Theta 2.50

3. How many pounds of apples must Chris pick before Pam's orchard is cheaper than David's?

   Show your work. ____________________

Source: Mathematics Assessment Resource Service (MARS). Reproduced with permission.

Note: Only Item 3 from Picking Apples pertains to this progression. The remaining items 1 and 2 do not occur in this progression.