



An Up-Close Look at Student-Centered Math Teaching



A Study of Highly Regarded High School Teachers and Their Students

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INTRODUCTION

Today, far too many students see mathematics as a subject that must be endured, not a source of inspiration or a way of thinking that can enrich their understanding of the world. Advocates of student-centered instruction argue that good teachers have the ability to transform mathematics classrooms into lively, engaging learning environments in which students take charge of their own learning and make meaningful connections to the world around them. Many of the principles and the practices of student-centered instruction are informed by research, but there is still a lot to learn about student-centered teaching, especially in the context of high school mathematics. This report seeks to contribute to the research base on student-centered teaching in mathematics in two ways. First, through an in-depth case study and qualitative methods, the report provides a nuanced portrait of student-centered teaching, which reflects the different ways in which student-centered teaching occurs in high school mathematics classrooms and compares the philosophy, instructional environments, and student perspectives of teachers who implement student-centered approaches to varying degrees. Second, the report also uses quantitative methods to examine differences in engagement and problem-solving ability among students assigned to teachers who implement varying degrees of student-centered instruction. Thus, the results of the study are potentially useful for researchers and practitioners who are interested in understanding

student-centered teaching practices in high school mathematics more deeply, as well as the effects of these practices on students. ■■■

MATHEMATICS AND THE NEW GLOBAL ECONOMY

The 21st century economy is now here. Business leaders, policymakers, and researchers continue to highlight the economic importance of increasing the proportion of students who succeed in science, technology, engineering, and mathematics (STEM). They argue that the prosperity and the security of the United States are dependent on innovations in science, technology, and engineering, with mathematics as the common underlying language (National Mathematics Advisory Panel, 2008; National Research Council, 2007). This new economy requires workers who can make sense of and tackle complex problems, work on collaborative cross-disciplinary teams, use high-level mathematical reasoning skills and digital technology, and communicate their ideas clearly (Wilson & Peterson, 2006). Journalists are also spreading this message widely, emphasizing the increasing value of workers who are able to apply mathematical knowledge in a digital, data-driven global economy (Davidson, 2011; Friedman, 2007). According to the U.S. Department of Education's STEM website, demand for STEM-related jobs will increase by more than 50 percent by 2020.

“*Today, far too many students see mathematics as a subject that must be endured, not a source of inspiration or a way of thinking that can enrich their understanding of the world.*”

Domestic and international student performance data continue to demonstrate that our nation's capacity to meet the increasing demands of the STEM workforce is on shaky ground. Most recently, the United States ranked 36th out of 65 education systems and below the international average in mathematics on the 2012 Programme for International Student Assessment (PISA; PISA, 2013). In addition, barely one third of U.S. eighth graders scored proficient or higher on the 2013 National Assessment of Education Progress (NAEP; National Center for Education Statistics, 2013). ACT results for the high school class of 2011 indicated that only 45 percent of 12th-grade students who took the test demonstrated readiness for college-level mathematics (ACT, 2011). Earlier iterations of PISA and NAEP, as well as the results from the Trends in International Mathematics and Science Study (TIMSS), painted a similar, lackluster picture of U.S. mathematics achievement (Stigler & Hiebert, 1997; U.S. Department of Education, 2003).

Although we are now experiencing the 21st century global economy first-hand, mathematics education researchers and policymakers have been actively preparing for this moment for a long time. Based on similar data and input from business leaders in the early 1980s, the National Council of Teachers of Mathematics (NCTM) produced a series of influential curriculum and teaching standards designed to help prepare all students to succeed in the 21st century economy (NCTM, 1989, 1991, 2000). The new Common Core State Standards for Mathematics incorporate aspects of the NCTM's standards as well as common instructional principles of higher-performing countries identified in comparative international studies (National Governor's Association, 2010; NCTM, 2013). The Common Core State Standards for Mathematics, which have now been adopted by 43 states, emphasize that mathematical proficiency involves more than rote application of mathematical algorithms. Mathematically proficient students must also be willing and able to apply mathematics concepts and describe their reasoning,

formulate and solve problems, and persist when a solution is not immediately apparent. These are also the types of traits required to succeed in the STEM workplace (National Governor's Association, 2010). Thus, understanding how to create mathematics learning environments that foster these attributes is likely to remain a top priority for policymakers and practitioners for the foreseeable future.



WHY WE NEED TO KNOW MORE ABOUT STUDENT-CENTERED MATHEMATICS INSTRUCTION

Mathematics lessons in the United States have traditionally consisted of a teacher lecture followed by student practice, with a focus on the application of procedures (Hiebert et al., 2003; Stigler & Hiebert, 2004). The prevalence of this type of rote instruction, despite its widely recognized flaws, was an important driver behind the mathematics reforms of the 1980s and continues to drive the CCSSM and other reforms in mathematics education today. Educators will need to make significant instructional shifts to help students reach standards that emphasize not only application of mathematical procedures, but also deep understanding, problem solving, critical thinking, and communication. The instructional shifts associated with creating these types of learning environments for students reflect many of the principals of student-centered instruction.

The term student-centered does not refer to a single instructional method. Rather, student-centered learning consists of an array of complementary approaches to teaching and learning that draws from multiple theories, disciplines, and trends in the field of education (Bransford, Brown, & Cocking, 2000; Deci, Koestner, & Ryan, 1999; Dewey, 1938; Fischer, 2009; Hinton, Fischer, & Glennon, 2013; Moller, Deci, & Ryan, 2006; Murnane & Levy, 1996; Piaget, 1952; Rose & Meyer, 2002; Trilling and Fadel, 2009; Willis, 2006; Wilson & Peterson, 2006; Vansteenkiste,

Simons, Lens, Sheldon, & Deci, 2004; Vygotsky, 1978; Zull, 2002, 2011). As the term implies, these approaches place students at the center of the learning process, preparing them to be successful both inside and outside of the classroom.

Student-centered instruction often features:

- *an emphasis on knowledge and skills associated with traditional content areas (e.g., mathematics, science, English, and history) as well as 21st century skills (problem solving, critical thinking, and communication)*
- *instructional activities that actively engage students in sense-making by building on their prior learning and connecting to personal experience, often through collaborative group work.*
- *a learning environment characterized by trust and strong relationships between and among the teacher and students.*
- *a focus on the individual through differentiation, scaffolding, and opportunity for choice.*

By supporting students as they actively develop the knowledge and skills important to their success inside and outside of the classroom, student-centered approaches hold promise for preparing students to achieve more rigorous academic standards, such as the CCSSM, and expectations for work in the new global economy.

Although there is a growing body of research related to student-centered instruction in general,¹ less is known about whether and how particular principles of student-centered approaches apply to mathematics teaching, particularly at the high school level. In fact, some of

the limited research on principles of student-centered instruction in mathematics suggests that teachers who think they are implementing features of student-centered mathematics instruction may not actually be doing so (Cohen, 1990; Stigler & Hiebert, 1997; U.S. Department of Education, 2003). One notable example is the TIMSS video study, which illustrated that teachers who thought they were teaching conceptually and playing more of the role of facilitator were actually delivering procedural, “mile wide and inch deep” traditional instruction (Stigler & Hiebert, 1997; U.S. Department of Education, 2003). This finding suggests that further refinement and specificity of the principles of student-centered mathematics teaching is warranted.

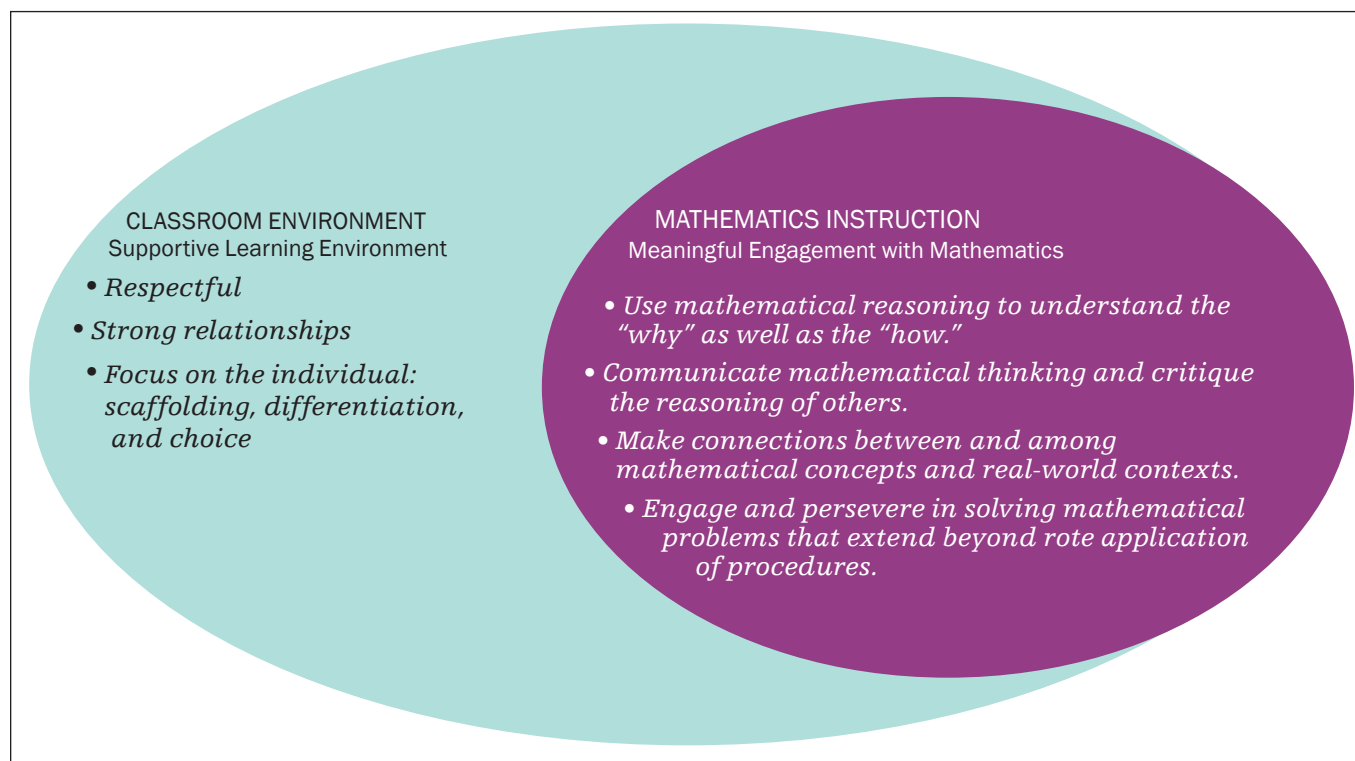
LENS FOR STUDYING STUDENT-CENTERED MATHEMATICS INSTRUCTION

The study team knew that it was important to develop a clear approach to studying and describing student-centered mathematics instruction, but specifying the array of practices and learning opportunities that comprise this type of instruction is complex. Some features of student-centered mathematics instruction apply to the general classroom environment, while other features apply to how students interact with the mathematics. As illustrated in Figure 1, student-centered classroom environments are characterized by mutual respect and trusting relationships between students and teachers. They are personalized in that the individual needs and interests of students are part of the classroom culture. Beyond these general classroom environment characteristics, student-centered mathematics classrooms provide opportunities for meaningful engagement with [the] mathematics. That is, the mathematics instruction provides students with opportunities to:

¹ For a concrete example of how student-centered teaching has been incorporated into a network of charter high schools, see Stephen and Goldberg’s (2013) profile of the High Tech High Network.

- *use mathematical reasoning to understand the “why” as well as the “how.”*
- *engage and persevere in solving mathematical problems that extend beyond the rote application of procedures.*
- *communicate their mathematical thinking and critique the reasoning of others.*
- *make connections between and among mathematical concepts and real-world concepts.*

Figure 1. Characteristics of Student-Centered Mathematics Classrooms



The distinction between the classroom environment and instruction is important, especially in the context of this study. Consider a supportive, student-centered classroom environment where norms of respect and trust have been established and students receive individual support to scaffold their learning inside and outside of the classroom. Within this supportive environment, the way in which students interact with the mathematics can take many forms. In one classroom, instruction might focus on rote application of mathematical procedures through teacher lecture followed by student practice, similar to the predominant procedural approach characterized in the

TIMSS video study. In another classroom, instructional activities might provide students with opportunities to explore, problem solve, reason, and communicate about mathematics. Both classrooms offer supportive learning environments, but the second classroom provides more opportunities for students to meaningfully engage with mathematics and, thus, is more student-centered.

Given the current push for more rigorous standards in mathematics and the accompanying shifts required in the teaching of mathematics, this study is particularly interested in examining differences in mathematics

instruction. Because supportive learning environments are part but not all of what constitutes student-centered mathematics instruction, we recruited and carefully studied teachers who had established supportive learning environments and were highly regarded by their peers. The teachers we selected were well liked by students and had been successful in helping students to do well in mathematics, as reported by instructional leaders with whom they had worked. (See Figure 2 for a list of the common characteristics of these highly regarded teachers.) Because these highly regarded teachers implemented different approaches to mathematics instruction, we

were able to examine and describe the degree to which their instructional methods were student-centered using the criteria described in Figure 1. That is, we were able to study deeply the extent to which students were provided opportunities to meaningfully engage in mathematics—to use mathematical reasoning to understand “why,” to communicate mathematical reasoning, to critique the reasoning of others, to make connections among concepts and to the real world, and to engage and persevere in problem solving. The following section on research design provides more details about our approach. ■■■

Figure 2. Characteristics of Study’s Highly Regarded Mathematics Teachers

- *Create respectful learning environments*
- *Establish strong relationships with students*
- *Are responsive, with clearly articulated strategies for helping students who are behind*
- *Have strong reputations of helping students succeed in mathematics*
- *Possess strong planning, organization, and classroom management skills*

RESEARCH DESIGN

Using a sample of highly regarded teachers and applying the lens for studying student-centered mathematics instruction previously described, we designed a mixed-methods study to answer the following four research questions:

- 1** What are different ways in which highly regarded high school mathematics teachers implement student-centered instructional practices?
- 2** How does their teaching philosophy and instructional environment relate to the degree to which they implement student-centered instruction?
- 3** What types of instructional approaches do students say help them succeed in mathematics?
- 4** Are there differences in student engagement and problem-solving skills that are associated with the degree to which student-centered instructional practices are implemented?

We used a case study to answer the first three research questions, which are intended to provide insight into the different ways in which student-centered approaches are implemented in high school mathematics classrooms, the

contextual factors that support or hinder their use, and how students react to those approaches. This information might be useful to practitioners who are at different stages of implementing or supporting the implementation of

student-centered teaching approaches in high school mathematics. We used quantitative methods and a larger sample of teachers to answer the fourth research question, which is intended to expand the research base examining the relationship between student-centered instructional practices in mathematics and student outcomes.

Sample and Selection

We drew our sample of teachers from six New England states and New York. The sample included seven participants for the case study and 22 participants (the original seven case study teachers, plus an additional 15) in the quantitative component of the study. Each teacher was highly regarded (see Figure 2), and they all represented a range of approaches to mathematics instruction.

Recruiting Highly Regarded Mathematics Teachers with Different Instructional Styles

Beginning in the spring of 2013 and continuing through the following summer, we reached out to district and school leaders, as well as representatives from student-centered school networks and organizations dedicated to promoting student-centered teaching approaches, to solicit nominees. To ensure that our applicants included a range of approaches that could be used for comparison to address the research questions, we collected and analyzed several types of data, described in Table 1.

We received 34 complete applications, which were carefully reviewed by a research team of three mathematics content experts with considerable high school teaching and leadership experience. For cases where the application was incomplete or required further clarification, we conducted follow-up phone interviews. Mirroring the lens for studying student-centered mathematics instruction previously described, we used two sets of criteria to determine which teachers were most appropriate for the study. One set of criteria focused on confirming that teachers were highly regarded and maintained supportive learning environments for students. The other set of criteria were used to determine whether teachers were more or less student-centered in their approach to mathematics instruction. The research team selected 26 highly regarded teachers representing different degrees of student-centered instruction to participate in the study.

All 26 teachers agreed to participate in the quantitative component of the study, but teachers had the option of participating in the case study, which was more intensive in terms of data collection. Sixteen of the 26 teachers elected to be considered for the case study, which required an in-person observation and brief interview with one of the project’s three mathematics education experts. The observation rubric included general features of high-quality instruction (e.g., the lesson was well prepared, students were on task, the presentation of the mathematics

Table 1. Data Sources Used to Classify Teachers

DATA SOURCES	INFORMATION
Teacher Application/Interview	approach to and design of mathematics instruction; ways of supporting struggling students
Instructional Leader Survey	degree to which they consider teacher to be “one of the best;” teacher’s approach to supporting struggling students; teacher’s instructional style; teacher’s ability to support student success
Classroom Observation (case study candidates, only)	implementation of mathematics instruction

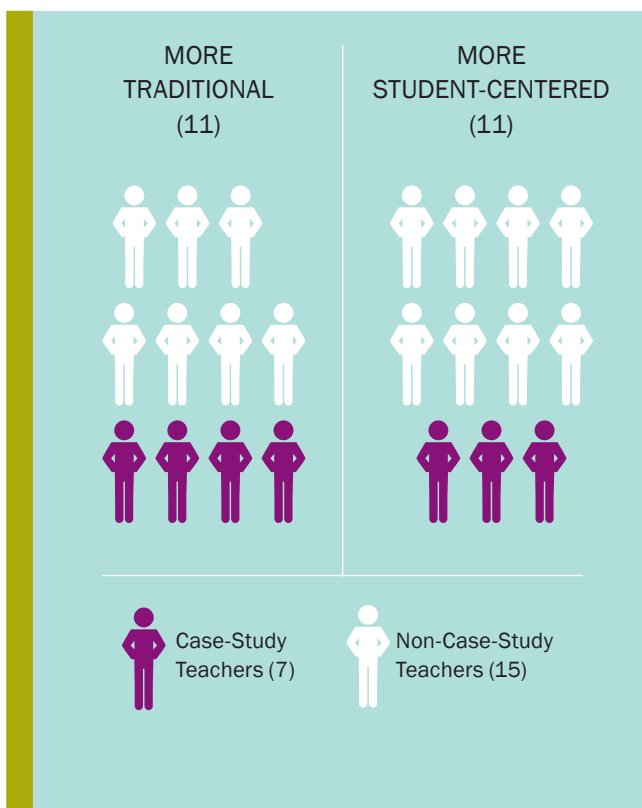
was clear and correct, and the classroom climate was respectful) and the degree to which instruction provided students opportunities to engage meaningfully with the mathematics. The team of three mathematics education experts reviewed the observation data and jointly settled on the final sample of seven case study teachers, who represented teachers with the highest quality lessons and varying degrees of student-centered instruction.

After the initial data collection activities concluded, 22 of the 26 teachers initially selected for the study remained; they included all seven of the case study teachers and 15 of the original 19 non-case study teachers.² In the interest of ensuring a range of instructional approaches, the research team examined the available data to see how many of these teachers tended to use more traditional or more student-centered approaches to mathematics instruction. Among the 22 teachers, 11 tended to implement more student-centered approaches to mathematics instruction (i.e. provided more opportunities for meaningful engagement with mathematics, as described in Figure 1), and 11 tended to use more traditional approaches. Among the seven case study teachers, four tended to use more traditional approaches to mathematics instruction and three tended to use more student-centered approaches (see Figure 3).

It should be noted that this initial grouping was done to ensure that we had a sample of teachers that represented enough variation in mathematics instruction. It does not suggest that a given teacher never used one or the other approach. In fact, in reviewing the data, the team found examples where teachers used varying degrees of student-centered approaches at different points in the lesson and in different contexts. The grouping presented in Figure 3 reflects the approach that the teacher implemented with the most regularity according to the data sources collected

during recruitment. The grouping presented in Figure 3 reflects the approach that the teacher implemented with the most regularity according to the data sources collected during recruitment.

Figure 3. Study Teachers' Approaches to Mathematics Instruction, Initial Grouping



² One of the case study applicants decided not to participate in the non-case study portion of the study when she was not selected for the case study. Another non-case study teacher did not return from maternity leave in time for data collection. Two additional teachers did not complete the study.

Sample

The study's final sample of 22 teachers varied in terms of background and professional context. Characteristics of the teachers and their schools are shown in Table 2.

Table 2. Demographics and Professional Context of Participating Teachers, by Initial Grouping According to Instructional Approach

CHARACTERISTICS OF TEACHERS	OVERALL (N =22)	TENDING TO USE MORE STUDENT-CENTERED APPROACHES (N=11)	TENDING TO USE MORE TRADITIONAL APPROACHES (N=11)
GENDER (%)			
Male	27.3	27.3	27.3
Female	72.7	72.7	72.7
HIGH SCHOOL TEACHING EXPERIENCE (%)			
1-3 years	13.6	9.1	18.2
4-10 years	18.2	9.1	27.3
11 years or more	68.2	72.7	63.6
DEGREE IN MATHEMATICS OR MATHEMATICS EDUCATION (%)			
Bachelor's	63.6	63.6	63.6
Master's	27.3	27.3	27.3
SCHOOL LOCATION (%)			
Large or small city	22.7	27.3	18.2
Suburb	22.7	18.2	27.3
Rural or town	54.5	54.5	54.5
SCHOOL ENROLLMENT (%)			
Fewer than 300 students	13.6	18.2	9.1
300-999 students	68.2	63.6	72.7
1,000 or more students	18.2	18.2	18.2
SCHOOL DEMOGRAPHICS (MEAN %)			
Minority	22.8	22.2	23.4
Free or reduced price lunch	30.0	35.7	24.4
Source: Common Core of Data, public and private school data 2010-2011, 2011-2012 school years, state department of education websites, and study records. Note: Gender, high school teaching experience, and degree in mathematics or mathematics education were collected as part of the study's teacher survey.			

About three-quarters of the 22 teachers were female and one quarter were male. They had an average of 16 years of experience, although five of the teachers had been teaching for fewer than five years. Eighteen of these teachers had an undergraduate degree in mathematics, and eight held a master's degree in mathematics or mathematics education. These numbers were similar for the teachers who tended to implement more or less student-centered approaches, although the teachers who tended to use

more student-centered approaches were slightly more experienced.

In terms of the school context, all but a few of the teachers taught in regular public schools. A few schools were public charter or magnet schools, and a few schools were private schools. The schools were located across the six New England states (Vermont, Maine, Rhode Island, Massachusetts, Connecticut, and New Hampshire) and

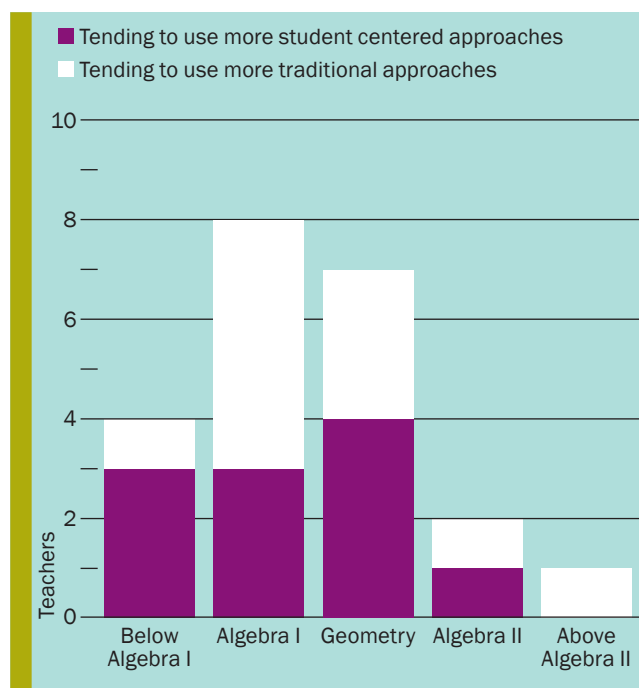
New York and were situated in a wide range of geographic settings. About one half of the schools were located in rural or small town jurisdictions; the other half were located in suburban, small city, or large city settings.³ The average number of students per school was approximately 650, with the smallest school enrolling 115 students and the largest enrolling more than 1,500 students. The percentage of students from minority families and students eligible for free or reduced-price lunch across all study schools was 22 percent and 30 percent, respectively. On average, schools participating in the study were somewhat smaller and less racially or ethnically diverse than national averages, but they were representative of their host cities or states.⁴ Like the teacher characteristics, the school characteristics were similar for teachers representing different instructional approaches. The only exception is that the schools of teachers who used more student-centered approaches had a higher proportion of free and reduced price lunch students (about 36%) than those that did not (about 24%).

Target Class

For each teacher, the research team chose a target class to serve as the focus of the study. To match the age range of students who take the Programme for International Student Assessment (PISA), from which the study's problem-solving assessment was comprised, each target class had to consist of a majority of 15- and 16-year-old students. This meant that the overwhelming majority of students were in either grades 9 or 10 and were enrolled in Algebra I, Geometry, Algebra II, or an integrated equivalent.⁵ The types, the difficulty level, and the range of mathematical subjects taught by the teachers in the study were

comparable across the two groups of teachers (i.e., those classified as more student-centered versus those identified as more traditional in their approach to mathematics instruction). Figure 4 shows the target classes included in the study, broken out by whether the teacher tended to implement more student-centered or more traditional approaches to mathematics instruction.

Figure 4. Full Sample of Study Teachers' Target Mathematics Classes, by Approach to Mathematics Instruction



3 These numbers were taken from the most recently available local education agency universe and public elementary/secondary school universe data files released as part of the Common Core of Data, a program of the U.S. Department of Education's National Center for Education Statistics that annually collects fiscal and non-fiscal data about all public schools, public school districts, and state education agencies in the United States.

4 Nationally, the average high school enrollment is 684 students, the overall minority percentage is 48 percent, and the percentage of high school students eligible for free or reduced-priced lunch is 38 percent. For the six New England states, the average high school enrollment, the overall mean percent minority, and the mean high school percent eligible for free or reduced-price lunch are 704, 22 percent and 30 percent, respectively.

5 Integrated mathematics programs are three- or four-year programs that integrate Algebra I, Geometry, and Algebra II content within and across courses. They are often titled Math I, II, III, and so on.

Data Sources

To answer the study’s research questions, we collected several different types of qualitative and quantitative data from all of the participating teachers, students, and schools. We collected information regarding school context, teacher and student characteristics, student performance, and student engagement. For the seven case study teachers, we collected additional sources of

data to more fully capture their philosophy of mathematics instruction, approach to planning, and enacted practice, as well as students’ perceptions about mathematics teaching and what helps them be successful. The data sources, the nature of the data, the target classrooms, and the associated research questions are presented in Table 3. All of the data were collected during the 2012–13 school year. ■■■

Table 3: Qualitative and quantitative data from all participating teachers, students, and schools

DATA SOURCE	NATURE OF DATA	TARGET CLASSROOMS	RESEARCH QUESTIONS
videos of mathematics instruction	<i>instructional practices implemented in lessons where a new mathematics concept is introduced (three lessons per teacher)</i>	case study teachers	1
instructional logs	<i>description and examples of instructional activities used throughout a week of instruction (one week per month per teacher for eight months)</i>	case study teachers	1
teacher interview	<i>teachers’ perceptions of their school and mathematics department, philosophy of mathematics instruction, planning process, instructional practices, and the challenges faced in implementing instruction aligned with their philosophy</i>	case study teachers	2
student focus groups	<i>students’ perceptions of their experiences in mathematics class and the factors that contribute to student success in mathematics (three to five students per teacher)</i>	case study teachers	3
administrative records	<i>demographic data (e.g., the percentage of students from minority families, English language learners, students in special education, and students who receive free or reduced-price lunch as well as grade 8 achievement on state mathematics tests) at the student and school levels</i>	all teachers	2, 4
teacher survey	<i>frequency of instructional practices implemented with the target class</i>	all teachers	4
challenging assignments	<i>examples of the most challenging assignment (to be completed by individual students) offered to the target class over a specified period of time</i>	all teachers	4
student survey	<i>students’ perceptions of their school and their experiences in the target mathematics class</i>	all teachers	4
mathematical problem-solving assessment	<i>student responses to publicly released items from PISA, an international assessment given to 15- and 16-year-old students</i>	all teachers	4

STUDENT-CENTERED APPROACHES TO MATHEMATICS INSTRUCTION

Our first research question explored the different ways in which highly regarded high school mathematics teachers implemented student-centered instructional practices. We examined video and instructional log data to identify the range of student-centered approaches implemented by our seven case study teachers. We present those findings in the context of two common parts of math lessons: the development of new mathematics and reinforcement of prior mathematical learning.

During the development phase, students are presented with and time is spent fully developing a new mathematics concept or rule. This can happen at any point in the lesson and often happens at the beginning of a lesson, after a review of homework, or after a warm-up problem. This type of instruction might happen more than once during a lesson and can take any amount of time to complete. In some cases, development activities can take the entire class period.

The reinforcement phase is when students have the opportunity to strengthen their understanding and practice applying mathematics content. Like the development of new mathematics, reinforcement opportunities can occur at any point in the lesson—during the warm up, homework review, classwork, homework—and may appear at several points in any lesson.

In our examination of these two parts of a lesson, we focused on the types of learning activities offered, as well as the discussion and other forms of communication

surrounding those activities. As expected from our recruitment efforts, our analyses showed varying degrees of the mathematical learning opportunities outlined in Figure 1. To make these distinctions concrete, we present a series of brief vignettes with accompanying discussion of the student-centered features in each example.

Development of New Mathematics

The most commonly implemented structure for the development of new mathematics was a teacher-led whole-class discussion. Within this structure, the teacher would generate interest in the new concept with a brief introduction and would then guide students through a sequence of examples and associated questions that, when completed, would lead to the new mathematical rule, procedure, or concept. Although the general structure appeared similar across many lessons, we observed some variety in the degree to which specific activities and discussion techniques engaged students in mathematical thinking.

The biggest difference that we found in approaches to whole-class discussion was the degree to and way in which students contributed. The following two vignettes illustrate that difference. In the first, the rules for solving absolute-value equations are being developed through a teacher-guided discussion, with some student contribution. In the second vignette, the relationship between two secants is developed through teacher-guided discussion, but with stronger student contribution. After we discuss these two teacher-guided vignettes, we present a third vignette that represented a less common approach to developing new mathematics: active student exploration.

Vignette 1: Teacher-Guided, with Some Student Contribution

TEACHER: Okay, we are going to start with our basic definition of absolute value before you put it into an equation, before you try to solve it, and before you get an application. The absolute value of a is the distance of a from zero on a number line. So we are talking about the distance from zero. Every time I teach basic absolute value, I love to talk about Ferris Bueller. I want you think for a minute: Distance is always positive. So why would I think about Ferris Bueller if I am thinking about distance and I'm only thinking about it being positive?

STUDENT: He tries to drive the car backward.

TEACHER: Why does he try to drive the car backward?

STUDENT: Because he thinks it will remove the miles.

TEACHER: He thinks it will remove the miles. So he's taken his best friend's dad's car...they go on a drive...and the valet at the parking garage puts more miles on it than should be. So they get it home, they prop it up, they stick the accelerator down, they put it in reverse...thinking they would be taking off the miles. Does it work?

STUDENT: No.

TEACHER: No, the basic thinking is that distance is positive. If I am here and I take three steps forward [models by taking three steps forward], how far have I traveled?

STUDENT: Three steps.

TEACHER: Three steps. Now, if I decided instead to turn around and take three steps backward (I am here and I walk three steps backward [models by starting at the same place and taking three steps walking backward]), how far have I traveled?

STUDENT: Three steps.

TEACHER: Still three steps. So it doesn't matter which direction I travel in. So from zero, it doesn't matter if I go negative or positive. That distance we always call positive, and that's why when we talk about absolute value, we talk about absolute value as always being positive. If I take the absolute value of x equal to 4 [writes $|x| = 4$], this means that our x is a distance of 4 away from zero. If that is to be true, what values are 4 away from zero on the number line?

STUDENT: Negative 4 and 4.

TEACHER: Negative 4 and positive 4. We can also call that plus or minus four—kind of a nice, short way to write it—then it represents the positive and the negative. Question?

STUDENT: Can absolute value be the distance away from another number, like one or two?

TEACHER: It's actually based on zero from a number line when it is just x [points to the x in the equation]. We are going to make these more complex and then you are going to see, especially when we get into an application, that it is going to look like it is distance from a different number.

The teacher then asks students to draw three number lines in their notebooks and explains that they will solve three basic equations. She begins with $|x| = 3$ and asks students to tell her the solutions. A student responds with "3 and negative 3." The teacher shows students how to represent those solutions on the number line and summarizes by saying that $|x| = 3$ gives them two solutions. She points out that, up until this point, they have not seen equations that have more than one solution, so this is something new for them. She continues with the next example, which is $|x| = 0$.

TEACHER: What should I plot on my number line now as possible solutions?

STUDENT: Zero.

TEACHER: Zero. Anything else?

STUDENT: No.

TEACHER: No, there is just zero. So how many solutions?

STUDENT: One.

TEACHER: Just one solution.

The teacher continues by asking students what she might choose as her third absolute-value equation. A student responds “negative x.” The teacher says it is not negative x. A different student suggests “x equals negative something.” The teacher takes this suggestion and asks students for the number of solutions to $|x| = -1$.

STUDENT: I feel like the positive one had two, then negative one would also have two.

TEACHER: Okay, so that is one idea. So one idea is that it should have two. Any other ideas?

STUDENT: Doesn't absolute value have to always be positive?

TEACHER: Absolute value has to always be positive. So how many solutions would I have?

STUDENT: None?

TEACHER: None...The whole idea of distance always being positive, you can never have an absolute value equal to a negative. So any time we come across an absolute value equal to a negative, we must say there's no solution.

At this point, the following rules have been established: (1) when the absolute value is equal to a positive number, you have two solutions; (2) when the absolute value is equal to zero, you have one solution; and (3) when the absolute value is equal to a negative number, you have no solutions.

In vignette 1, we see several practices that provide students with opportunities to meaningfully engage with mathematics. As the new material is being introduced, students make connections between absolute value (something they already know) and solving absolute-value equations. They also make connections to personal and real-world experiences through a discussion about the movie *Ferris Bueller's Day Off* and through the walking demonstration. These connections provide the reason why absolute value is positive: because it is associated with distance, not direction. As the development of new mathematics continues, the students are asked to come up with the answers to basic absolute-value equations. As they provide those answers, they discuss the reason for the answer—always within the context of distance. Throughout this lesson segment, the focus is on not only the rules, but

also the underlying reasoning that supports those rules. Students are offered the opportunity to communicate their thinking as they respond to questions from the teacher.

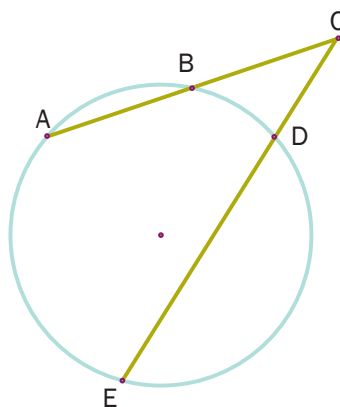
Although the discussion presented in this vignette provides some opportunities for students to engage meaningfully with the mathematics (and thus, illustrates some student-centered approaches to mathematics instruction), these opportunities were limited. Students were rarely presented with opportunities to communicate mathematical reasoning, critique the reasoning of others, or solve problems that extend beyond rote application of procedures. Vignette 2 illustrates the way in which activities and questioning could have been implemented within this teacher-guided structure to provide more of such opportunities.

Vignette 2: Teacher-Guided, with Strong Student Contribution

The introduction begins with the teacher asking students to work individually on the following task:

Tasks/Questions:

1. What are segments/lines AC and EC?
2. Draw chords AD and BE.
3. Are triangles ADC and EBC congruent? Explain your reasoning.
4. Are triangles ADC and EBC similar? Explain your reasoning.



As the students finish the task, the teacher engages them in a conversation.

TEACHER: What do we know about these two triangles (referring to triangle ADC and triangle EBC)? Can we prove the two are similar? Or are they congruent? From looking at them? Are these triangles congruent?

STUDENT: No

TEACHER: Why not? Why don't you think they are congruent? Do they look like they are exactly the same?

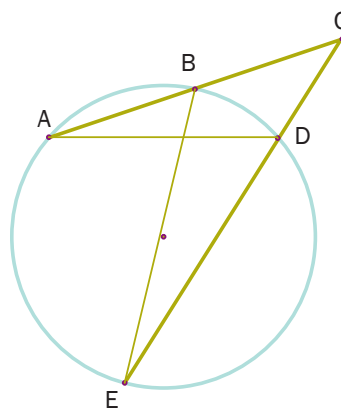
STUDENT: No.

TEACHER: No, they don't look like they're exactly the same. They don't look like they are congruent, but let's think about what we do know about these triangles. So what do we know about these two triangles? Do they have anything in common? Anything similar about them?

STUDENT: They have two angles in common: Angle C and angle BAD and angle BED.

TEACHER: How do we know that?

STUDENT: Both of the triangles include angle C in them, and angle BAD and angle BED both open up to the same intersecting arc.



The teacher projects a picture of the figure on the board and demonstrates, visually, what the students were saying.

TEACHER: What type of angles are these? What do we call angles with the vertex on the circumference?

STUDENT: Inscribed angles?

TEACHER: Yeah, and to find the measure of the inscribed angle what would we do? To find the inscribed angles, how would we do that? What would we do?

STUDENT: They have the same arc.

TEACHER: Exactly, so if they have the same intercepted arc, must they be the same measurement?

STUDENT: Yes

TEACHER: Yeah. We want to prove that the two triangles formed here are similar, using what we just discussed.

The teacher then leads the group through a discussion of how to prove that the two triangles are similar. The teacher notes that there is no singular way to start the proof and prompts students to supply statements and justifications that, when put together, create a logical argument to prove that the triangles are similar. Throughout, she asks students whether or not they disagree with pieces of the proof offered by one of their classmates. After two pieces of the proof have been offered, she asks them if there is enough information to prove the two triangles are congruent.

TEACHER: Who thinks that it is enough information to prove the two triangles are congruent? [waits for hands to be raised] Nobody? Who thinks that is it not? [sees hands being raised] Why not? Who would like to tell us?

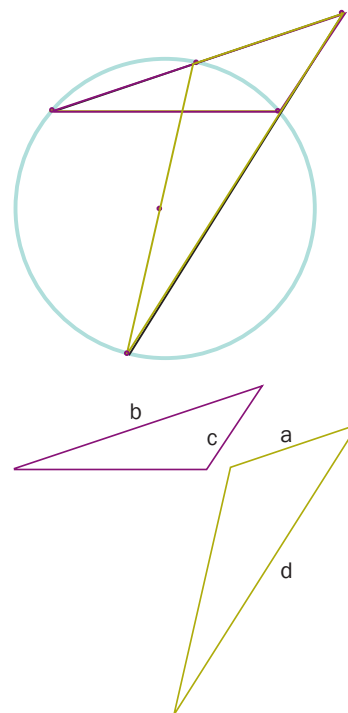
STUDENT: We have the equation that they are similar, not congruent.

TEACHER: How come?

STUDENT: Because we only know angles; we don't know side lengths.

The teacher then uses questioning to guide the students to finish proving that the two triangles are similar. She then asks what they know about similar triangles. The students tell her that sides are proportional. The teacher then provides a visual diagram and asks students to write the associated proportion, which is then manipulated to result in the relationship between the segments formed by two secants and a circle.

At this point, a rule has been established regarding the relationship between segments of secants that intersect in a point outside of the circle. The teacher summarizes by saying the rule is "whole times the outer part = whole times the outer part."



$$\frac{c}{a} = \frac{b}{d}$$
$$a \cdot b = c \cdot d$$

Like the first vignette, the introduction of a new mathematical concept in the second vignette provides opportunities for students to connect the new mathematical idea with ideas they already know, namely secants and similar triangles. Similarly, like in the previous vignette, there is a strong focus on the underlying reasoning that supports the new mathematical relationship. In both cases, questioning is used to invite students to communicate their thinking and contribute to the development of the new mathematical concept.

In contrast with the first vignette, however, where only a few students answered questions, the introduction in the second requires all students to investigate the relationship between the two triangles. Both approaches provide opportunities to engage, but by requiring all students to complete an exploration, the second has more potential to draw all students into thinking about the new ideas.

The second vignette also provides more opportunities for students to communicate their reasoning about the new mathematical idea. In the first vignette, students were expected to reason about individual problems that had a specific solution. In the second, students were expected to build on their classmates' contributions to develop a proof that would ultimately lead to the new mathematical relationship that was the focus of the lesson. As the teacher mentioned, there was no singular way to construct that proof. In contrast to the activity in the first vignette, this

activity allowed for different solution paths and involved much more than rote application of a procedure.

In addition to the activities themselves, the kinds of questions posed to students were different. In the first vignette, students were expected to provide the answer, but in the second, students were expected to provide the answer as well as the reasoning behind their answer. The teacher in the second vignette often followed a student response with “How do we know that?” and “How come?” Students in the second vignette were also asked to consider other students' thinking. When a student provided a step in the proof, the teacher would ask whether or not the rest of the class agreed.

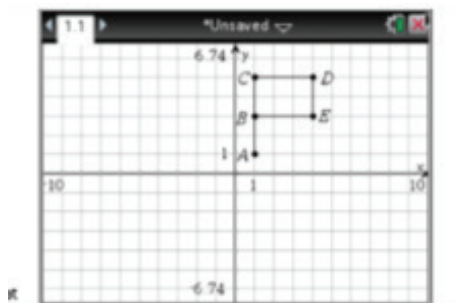
These two vignettes illustrate the ways in which, even within the common structure of teacher-guided discussion, a slight difference in the task or the way questions are posed can provide more opportunities for students to engage meaningfully with mathematics.

Although the teacher-guided approach was most common, we also found examples of a different approach that provided even more sustained opportunities for students to meaningfully engage with mathematical content. In this other approach, students engage in active exploration, with the teacher acting as a facilitator of discussion. The approach is illustrated in Vignette 3, where students are learning the rules for geometric transformations.

“ *A slight difference in the task or the way questions are posed can provide more opportunities for students to engage meaningfully with mathematics.* ”

Vignette 3: Exploration, with Strong Student Contribution

The introduction begins with students standing around the circumference of the room. The desks are moved out of the way, and a grid is marked on the floor. The teacher reminds the students that they have been working with shapes on the coordinate system. She tells them that they will now take those shapes and move them around. They will come up with rules that govern all of those things. She asks for five volunteers to create the following shape on the floor.



Each volunteer receives a nametag with an A, B, C, D, or E on it—to represent the points on the shape. The students work together to place the volunteers at the correct places. The teacher tells them that they are the “pre-image” and asks “What happens if our flag moves three units to the left?”

The teacher asks for five more volunteers, and the students work together to place those volunteers in the appropriate place to represent a shift three units to the left.

TEACHER: Take a look at the picture, is it right? What did we do? We took the original flag... and I asked you to move the original flag three units to the left. So, you guys think about where you are [motioning to the image points]...are you, image points, three units to the left of your original?

STUDENT: Lauren, you are not three points away from Tyler and Brian. You guys have to be over here.

STUDENT: No, I have to be three points from point B.

STUDENT: If I have to be three points from Tyler...

SEVERAL: You are over here.

STUDENT: So I'm at...

STUDENT: So where am I?

STUDENT: You are right here.

This conversation continues as students move to new (correct) locations. The teacher double checks that everyone is in the right place. The students again discuss among themselves to be sure they all agree. Meanwhile, the students who are not volunteer points serve as data collectors and record the coordinates of each of the image points.

TEACHER: What is the difference between the original flag and the image flag, the pre-image, and the image? [students discuss among themselves] Are they different shapes?

STUDENTS: No.

TEACHER: How do you know?

SEVERAL: Because you just slide it.

The teacher thanks the volunteers and asks for other volunteers to do the next figure. This time, the image is four units below the pre-image. As with the previous example, the students work together to place the volunteers, the teacher asks them to double-check their location, the students work together to check, and the data collectors record the points. This process repeats for movement three units left and two units down. The students then gather to be sure everyone gets the points from the data collectors, and the teacher then puts them into groups.

TEACHER: What we are after are called coordinate rules for these transformations... I want you to think about what is happening to those coordinates. A coordinate rule looks like “x, y goes to, or becomes,” and then two new coordinates [writes $(x,y) \rightarrow (\quad , \quad)$]. So you describe what did I do to x or what will I do to x and what do I do to y. Okay? So by analyzing the coordinates that you have from the original to this one [points to the final one on the page], see if you can come up with a coordinate rule that describes how you can go from the original flag, the pre-image, to the image flag. Think about that to yourself first. See if you have an idea and then you can talk to a partner or a little group. Okay? And compare.

Students begin to discuss in small groups. The teacher tells the class that the rule should still have “x’s and y’s” in it because “it is a general rule.” As she circulates, she reminds each student of this.

TEACHER: Here is what I see a lot of as I walk around the room. [She writes $(x,y) \rightarrow (-3,0)$] I get what you are trying to say. I think what you are trying to say is that the x went left by negative three and the y stayed the same. What this rule actually says is that x becomes negative three, regardless of what it is to begin with, and y becomes zero, regardless of what it is to begin with. So think about that. Think about what adjustments you want to make to this and talk to each other.

The students talk in small groups. They share ideas, ask questions of each other, and try things out. The teacher encourages them to continue by looking at all three rules and, finally, asks them to come up with a rule for a transformation that moves h units horizontally and k units vertically.

The lesson continues by following the same process for developing coordinate rules for reflections and rotations. When the students have finished exploring, the teacher leads a whole-class discussion in which they state the final rules. As they shared their rules, the students argued among themselves about the correct way to state the coordinate rule.

In this vignette, students provided most of the reasoning that supported the development of new mathematical knowledge. They did so by engaging collaboratively in exploratory activities that were carefully designed to support investigation, by looking for patterns, and by coming up with a general rule. Throughout, the teacher did very little of the reasoning. At strategic times, particularly when she noticed students were not reaching the desired answer, she asked questions to push students' thinking and allowed them to argue about the answer amongst themselves. She did not tell them the answers and allowed them to persevere.

Through the choice of task and use of questioning, this third vignette provides a strong example of how teachers can implement activities that engage as many students as possible in reasoning about mathematics, communicating mathematical thinking, and persevering in problem solving. In this particular case, there were no explicit connections to previously learned mathematics and/or personal experience. However, by allowing students to physically enact the transformations, this development provides a personal experience upon which students can draw as they apply the rules to new examples. Taken together, the learning opportunities illustrated in this vignette are more strongly student-centered than the first two vignettes.

Reinforcing Mathematics Learning

Another core element of mathematics instruction is reinforcing prior mathematical learning. The most commonly used approach to reinforcing mathematical learning in this study was providing students with mathematics problems to solve, individually or in groups. As students completed the problems, the teacher typically would circulate, monitor student progress, and answer questions. Often the teacher would, at strategic points in the lesson, pull the whole group together and discuss the answers to the problems. Within this general structure, we found differences in the

degree to which the problems and the discussions engaged students in mathematical thinking.

Mathematics Problems

In many of the cases we observed, students were offered problems that required rote application of the mathematical procedure or rule discussed in the introduction and development phase of the lesson. Figure 5 illustrates two types of problems that illustrate the rote application of procedures related to linear systems of equations, a common topic in algebra.

Figure 5. Rote Application Problems

SOLVE BY GRAPHING.

- | | |
|---|---------------------------------|
| ① $y = -x + 2$
$y = -\frac{1}{2}x + 1$ | ③ $y - 2x = 1$
$2y - 4x = 1$ |
| ② $y = 3x - 3$
$y = x + 1$ | |

SOLVE BY SUBSTITUTION.

- | | |
|------------------------------------|----------------------------------|
| ④ $-5x + 3y = 12$
$x + 2y = 8$ | ⑥ $y + 5x = -3$
$3y - 2x = 8$ |
| ⑤ $x - 4y = 22$
$2x + 5y = -21$ | |

SOLVE BY ELIMINATION.

- | | |
|-------------------------------------|--------------------------------------|
| ⑦ $-3x + y = 7$
$3x + 2y = 2$ | ⑨ $-4x + 5y = -11$
$2x + 3y = 11$ |
| ⑧ $3x + 4y = -1$
$-9x - 4y = 13$ | |

A drummer is stocking up on drumsticks and brushes. The wood sticks that he buys are \$10.50 a pair, and the brushes are \$24 a pair. He ends up spending \$90 on sticks and brushes. He bought 6 pairs total. How many of each pair did he buy?

Problems such as these do not provide a great deal of opportunity to engage students in reasoning about the “why” or the “how,” communicate their thinking, critique the reasoning of others, or make connections to other

mathematics. For the most part, these problems require students to apply a rote set of rules or procedures to reach a solution. The drummer problem does provide an opportunity to make connections between the mathematics and real-world contexts. By situating the work in a real-world context and requiring the student to generate the equations needed to solve the problem, the drummer problem also has the potential to engage students in making meaning of the mathematics. However, as students continue to solve similar problems with the same structure, potentially situated in different contexts and/or with different numbers, these types of word problems can become proceduralized and solved mechanically.

In contrast, we found other examples of problems in the case study classroom that provided more opportunity for students to meaningfully engage with mathematics and, thus, would be considered a more student-centered approach to reinforcing student thinking. Figure 6 illustrates two of these examples, which are also related to linear systems of equations.

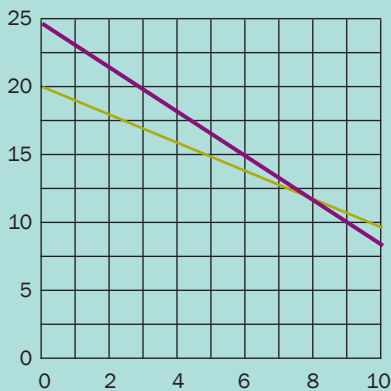
In stark contrast to the first set of problems on the same topic, these problems do not simply require rote application of a mathematical procedure. To solve these problems, students are required to think about the reasoning that underlies the mathematical procedure

Figure 6. Problems Requiring Reasoning, Critical Thinking, and Communication

Changes in the cost of the telescope eyepiece and the number of club members willing to work required solving the system $16x + 10y - 245$ and $x + y = 20$, where x and y represent the number of workers on outdoor and indoor work, respectively.

Robin produced this graph to use in estimating the solution. She estimated that $x = 7.5$ and $y = 12.5$ was the solution.

- a. Is that an accurate estimate?
- b. Does the solution make sense in the problem situation? Why or why not?



Faced with the following system of equations, two students, Lincoln and Claire, both decided to use the substitution(s).

$$\begin{cases} 5x - y = -15 \\ x + y = -3 \end{cases}$$

Lincoln's method
 $x = -3 - y$

$$\begin{aligned} \text{So, } 5(-3 - y) &= -15. \\ -15 - y - y &= -15 \\ -15 - 2y &= -15 \\ -2y &= 0 \\ y &= 0 \end{aligned}$$

So, $x = (0) = -3$.
 $x = -3$
 THE SOLUTION IS $(-3, 0)$.

Claire's method
 $y = -3 - x$

$$\begin{aligned} \text{So, } 5x - (-3 - x) &= -15. \\ 5x + 3 &= -15 \\ 4x + 3 &= -15 \\ 4x &= -18 \\ x &= -4.5 \end{aligned}$$

So, $(-4.5) + y = -3$.
 $y = 1.5$
 THE SOLUTION IS $(-4.5, 1.5, 0)$.

There are errors in the work of both Lincoln and Claire, but one of them was "lucky" and got the correct solution.

- a. What are the errors in each case?
- b. Which student got the correct solution? How do you know?

involved. In particular, they are expected to critique others' reasoning and provide a justification as to why they think the approaches used are correct and/or make sense. The telescope problem is an example of how critiquing a solution can leverage connections to underlying concepts and the real world. Both the drummer problem (Figure 5) and the telescope problem (Figure 6) are set in real-world contexts, but the telescope problem requires students to do more than write and solve a system of equations to model a real-world context, which could become a rote process. In the telescope problem, the students are presented with a solution and asked whether it makes sense in the problem situation. Thus, the telescope problem has a greater potential to involve students in making the connection between the context and the mathematics in a critical way.

Mathematical Communication

It was not only the problems themselves that provided different levels of opportunity for students to engage meaningfully with mathematical content, the discussions associated with the problems varied too. In instances where the teachers led the class to solve increasingly complex problems, we saw differences in the degree to which teachers asked students to reason about the mathematics and communicate their reasoning. In some cases, teachers merely asked students to provide the next step. In others, the teacher would ask students for the next step as well as the reasoning behind that step. These teachers would frequently ask "Why?" and "Can you explain?" as students provided the answers.

In instances where students worked on problems while the teacher circulated, we saw differences in the ways in which teachers interacted with the students, particularly when students were struggling with the material or had gotten an incorrect answer. In most cases, the teacher would scaffold their thinking, often through a series of short, closed-ended questions that would lead them to the correct answer.

Consider the following excerpt from a lesson on simplifying radical expressions, where the student is struggling to simplify $\sqrt{8}$:

Vignette 4. Scaffolding Student Thinking

TEACHER: What multiplies to 8? 1 times 8 and 2 times 4, right? So, if I go with 2 times 4, which one of those is the square?

STUDENT: Four.

TEACHER: So that's the one I can square root. When I square root that four, what does it become?

STUDENT: Two.

TEACHER: Good... and then what's left inside still.

STUDENT: Two.

TEACHER: Two. That four is like transformed and escaped.

This approach highlighted in Vignette 4 engages students in reasoning about the mathematics, but in a very pointed way. Throughout the interaction, the teacher is modeling the reasoning and the solution path and asking the student to contribute with closed-ended, factual questions. By connecting the discussion to what the student knows, the approach also has elements of student-centered instruction. However, it does not provide a great deal of opportunity for students to share her own thinking.

Compare this approach to another one where the teacher plays a quite different role in working with struggling students. In this excerpt from a lesson on scatter plots and regression, the students have been given some data on the hip angle and height of a set of horses. They are supposed to create a scatter plot of this data. As they do, they begin to question which variable—hip angle or height—should be represented on the horizontal axis, or x-axis.

Vignette 5. Allowing Students to Reason and Persevere in Solving Problems

STUDENT: Wait, what do we put on the bottom? Height or hip angle?

STUDENT: Height.

STUDENT: No, hip angle.

STUDENT: I think it is the hip angle.

STUDENT: Height is on the bottom.

STUDENT: No, because the independent is on the bottom.

STUDENT: No, there is no independent or dependent.

STUDENT: I put mine on the bottom.

STUDENT: Tyler, it is just like with the cars, the MPG is dependent on the weight, just like the height of the horse is dependent on the hip.

STUDENT: Height is not on the bottom.

STUDENT: So why would the height depend on the hip angle?

Each student makes a decision for himself or herself as to which variable to represent on the x-axis and creates the scatter plot on his or her own calculator. The scatterplots for all of the students are then projected for the whole class to see.

TEACHER: OK, interesting! Interesting! Joan, did you add your variables?

STUDENT: I don't know what to put for x.

TEACHER: Well, apparently there is not agreement, so do what you think is best, and we'll have a discussion about that. So I'm seeing some differences here in these scatter plots. So, I'm seeing a lot that look alike, but Jonas and Mike, yours looks different. What do you guys want to say anything about that?

STUDENT: (We put the) hip angle on the x-axis and the height on the y.

TEACHER: What was your reasoning behind that?

STUDENT: The height doesn't depend on the hip angle. The hip angle depends on the height.

TEACHER: If the hip angle depends on the height, then what's the dependent variable?

STUDENT: The hip angle.

TEACHER: And on which axis do we plot the dependent variable [discussion among students in the class]? So what you are telling me is that hip angle is a function of height. If you are telling me that hip angle depends on height, you're telling me that hip angle is a function of height. The inputs are the heights and the hip angles are the outputs? Is that the relationship?

STUDENT: Yeah.

STUDENT: If you put it that way.

TEACHER: So, if that's true, and I'm not saying it is, but if that's true, what goes on the x-axis? What's the independent variable?

STUDENT: Height.

STUDENT: How is height on the bottom?

TEACHER: If you make the assumption that hip angle depends on the height, then you need to be plotting height on the x-axis, on the horizontal axis because that's where the independent variable goes. Is it valid to make that assumption?

STUDENTS: Yes

TEACHER: Why?

STUDENT: Why not? [students laugh nervously]

TEACHER: So my question is... why you think it is reasonable to make that assumption?

STUDENT: Could it also be that the height depends on the angle?

TEACHER: That's part of my question.

Students discuss the question among themselves. As they do, the teacher reminds them of a discussion they previously had about mileage and weight of a car and the decision they made as to which was considered the independent variable and which was considered the dependent variable.

TEACHER: Is there any line of reasoning that height has to be the independent variable here?

STUDENTS: No.

TEACHER: Is it possible that hip angle could be plotted on the x-axis?

STUDENT: Yeah, because it is.

TEACHER: Does it make sense to do that?

STUDENT: It could.

TEACHER: So, I actually want to leave these guys like that and see what it does. Let's see what it does to the line in the correlation value if we switch our x and y. Okay? So, why don't you guys go ahead and put a regression line on your data.

Although both approaches provide some opportunity for students to engage meaningfully with the mathematics, the second does so to a greater degree. In contrast to the first example, the teacher does not immediately provide a series of closed-ended questions to point the students in a particular direction. Instead, the students are left to persevere and, in so doing, argue productively among themselves. The students are asked to communicate their own thinking and critique the reasoning of others. When the teacher does step in, she still does not provide a solution path. Instead, she facilitates a discussion in which students are expected to justify their thinking. She asks a series of open-ended questions and challenges the responses. At some point in the discussion, the teacher does provide an opportunity for students to connect the current work to work done previously, but she is careful to direct their attention to this particular real-world situation and challenges them to think about the particular context associated with this problem. Ultimately, the episode ends without a final answer. Instead, the students continue on to the next step, with different students doing different things and discovering what impact that choice might have as they move forward with the task.

Given the number of opportunities students have to engage meaningfully with the mathematics, the second example provides an example of a highly student-centered approach to addressing a concept with which students struggle. It could be argued that the context and nature of the scatter plot task lent itself well to the type of mathematical discussion we saw. While simplification of the expression requires students to apply a procedure, making a decision about how to plot a set of data so that it accurately represents the context requires a greater degree of critical thought. This distinction is important when considering lesson options for different mathematical topics. As students explore more complex and applied tasks that engage them in meaningful ways with the mathematics, there are likely to be more opportunities to facilitate mathematical discussions that do the same.

Summary

In looking across our observations of lessons that develop new mathematics and those that reinforce prior mathematical learning, we found several instructional techniques that provided all students with opportunities to engage meaningfully with mathematics that are outlined in Figure 1. These student-centered approaches, listed in

Table 4, fall into two categories: the instructional activities/tasks provided to students and techniques for orchestrating mathematical communication. These examples are discrete instances of how student-centered mathematics instruction

occurs and potentially fruitful techniques to explore with teachers who are interested in becoming more student-centered in their approach. ■■■

Table 4. Observed Student-Centered Approaches to Mathematics Instruction

CHARACTERISTICS OF INSTRUCTIONAL ACTIVITIES/TASKS	ORCHESTRATION OF MATHEMATICAL COMMUNICATION
Focus on the “why” as well as the “how”	Focus on the “why” as well as the “how”
Allow for multiple entry points and solution methods	Encourage students to justify and explain their solution strategies
Challenge students to reason about mathematics by looking for patterns, making conjectures, conducting explorations, examining connections between and among mathematical concepts, and justifying mathematical solutions/results	Encourage students to critique the mathematical reasoning of others
Make explicit the connections between mathematics and real-life experiences	Support students in advancing, but not taking over their thinking as they engage in a productive struggle with mathematics
Encourage the use of different tools, including technology, to explore mathematics and solve mathematics problems	Elicit and make connections between different mathematical ideas and/or approaches to the same problem
Provide opportunities for collaboration to communicate and critique mathematical reasoning	

FINDINGS PART TWO

LINKING INSTRUCTION, PHILOSOPHY, AND INSTRUCTIONAL ENVIRONMENT

Our second research question focused on the extent to which a teacher’s expressed philosophy and instructional environment relates to implementing student-centered approaches to mathematics instruction. To answer this question, we relied on data collected from classroom videos, instructional logs, and interviews with the seven case study teachers. In our analysis of instructional logs and videos, we noticed that none of the teachers implemented the same instructional approaches every single day, so for this analysis, we focused on differences in what could be considered typical practice (based on the available data).

As we examined typical practice for each of these teachers we noticed that, as expected, each of them exhibited common characteristics of highly regarded teachers. They all cared a great deal for their students and would go out of their way to provide extra support as needed – during class time, before school, during lunch, or after school. All of them believed that all students could learn mathematics, provided a classroom environment where their students felt safe and respected, and showed students how fun mathematics can be. They all tried to find ways to connect with students on a personal level and encouraged them to be successful. In short, they provided a very supportive learning environment for their students. Also as expected, when it came to instruction, the seven case study teachers did not fit neatly into the two groups

we initially created during recruitment. Rather, this particular set of teachers tended to fall into three groups: traditional with some student-centered features, a blend of traditional and student-centered, and strongly student-centered. We provide details about each of these three categories next.

Traditional, With Some Student-Centered Approaches to Instruction

Four of the case study teachers tended to use more traditional approaches to mathematics instruction, although they implemented student-centered approaches at times. To develop new mathematics, these teachers guided the reasoning, while providing some opportunities for students to contribute. Within this structure, these teachers would implement some student-centered approaches, such as making connections to previously learned mathematics, introducing a real-world context familiar to students, or engaging them in a hands-on exploration of the new mathematics. They would also focus the development phase of instruction on not just the rule, but the reasoning behind the given rule. However, when it came to the nature of student contribution during the mathematical development and reinforcing phases, these teachers implemented more traditional approaches. They typically asked students closed-ended questions, focused on the correct answer, and only occasionally asked students to justify their thinking. With the exception of an occasional problem that required mathematical reasoning and/or communication, problems and tasks used to reinforce and extend understanding tended to focus on procedures and application of a rule. In both the introduction and development and during the reinforcement and extension of student thinking, these teachers tended to scaffold student thinking with very pointed, close-ended questions. Students rarely had opportunities to critique each others' thinking.

When asked about their philosophy of mathematics teaching and learning, these four teachers said very similar things. They indicated a belief in the value of presenting students with a very structured environment and thought students learned best when presented with direct instruction, followed by a period of time to practice and tackle more challenging problems, usually in groups. One teacher explained, "Structure... I'm very structured. The familiarity allows them to feel more comfortable, and then there's not disruptions... There's no chaos. So, structure. I do direct instruction so that I can actually teach concepts, but then we do the application and challenge problems that stretch them."

To plan their lessons, these teachers reported using a variety of resources. Three of the teachers used a district-approved textbook as a primary resource for planning. The fourth teacher used resources developed through work done with a team of other teachers in the school as the primary resource; these resources were developed several years ago and were loosely based on the district-approved textbook. In all cases, the textbook referenced by the teachers was traditional in nature. Each lesson in the textbook focused on a particular mathematical rule or procedure, began with a brief description of the rule or procedure and the underlying concepts, included a series of examples to show how to use the procedure or rule to solve problems, and concluded with a series of practice problems. These practice problems represented a mix of rote application, word problems that, when repeated, can become mechanistic, and a few problems that required mathematical reasoning and/or critical thought.

To complement their primary curriculum resource, all four teachers indicated that they relied on a range of additional sources to help them plan lessons. They reported searching for interesting sources online and talking with others in the mathematics department at their schools. Three of the teachers described being a part of a team of teachers

who were responsible for the same course. This team of teachers was led by one teacher who was responsible for creating a calendar for each unit and common assessments to be given during that unit. As the team members worked through the unit, they would share worksheets and/or quick activities that they thought worked in their classrooms. They indicated that having such resources was useful in planning lessons.

Three of the four teachers also emphasized the value of high standards and of encouraging students to push themselves by taking responsibility for their own learning and working through difficult mathematics problems. Although they push their students to meet high standards, these teachers are aware that some students may need more support, particularly freshmen. They make sure to provide those supports. One teacher explained, “I believe that daily responsibility component is definitely important. I try to, especially with freshmen, train them in that because many of them have never studied for math before in their lives.” Another said, “One of the things I’ve pushed more this year is to get kids to want to do more on their own, not to come running to me or to somebody else. What can you actually do on your own?”

This focus on high standards with supports is evident in the context in which they teach. When asked about their school’s philosophy of teaching and learning, these three teachers cited a focus on passing courses and preparing for college. One teacher explained, “We expect the best from our students and...we definitely make sure that we get the best from our students.” All three of the teachers indicated that students in their schools have generally been very successful, and subsequent analyses of data released from each of the relevant state departments of education confirms their claims. On average, 84 percent of incoming students at each of these three teachers’ schools are proficient in mathematics. They are motivated and well

prepared. Their schools are focused on providing support to students who need it, and these teachers indicated that there are many ways students can get the support they need. Analyses of data from state educational websites support these claims.

Blend of Traditional and Student-Centered Approaches to Instruction

In contrast to the teachers who implemented predominantly traditional instructional approaches, two teachers tended to implement a blend of traditional and more student-centered approaches. To develop new mathematics, these teachers used a blend of teacher-guided, whole-class discussion and more exploratory approaches in which students worked in groups to find patterns, make connections, communicate their reasoning, and critique others’ reasoning. To reinforce prior mathematical learning, these teachers used a mix of problems that required rote application of procedures and problems that required more mathematical reasoning. While students worked, these teachers tended to ask pointed questions to scaffold student thinking and would, at times, ask students to communicate their thinking.

When asked about their philosophy of mathematics instruction, both of these teachers talked about the importance of a mix of exploration and practice. They both believed that, while some students will learn well through hands-on explorations, others may need more opportunity to practice applying procedures and focus on skill development. One of these teachers explained that she prefers to start with a mini, hands-on exploration but then allow students to shift to more traditional practice of procedures. She described her approach as follows:

“I will start out with the discovery and kind of see who it is working really well for, and then later that day or the next day we will have discovered all [mathematics rules] we will use and have to refer back to. So the students

who need that visualization, I'll continue to provide them with those [visualizations] to help them along, and for the students who that doesn't help, they're not required to use them."

The other teacher indicated that although she uses exploration to introduce and develop new material, she often starts by pre-teaching so that she "can remind them that this is not entirely new material or, even if it is, these are some things to look out for." After the exploration, she makes sure students have opportunities to practice applying newly acquired skills to a set of problems, particularly for homework, because she sees that "when kids leave the classroom, they forget what they've done and they find it hard to make that connection between this is what I just did in class and now this is a series of problems I'm going to do to kind of get that skill mastery."

The approved textbooks for one of the teachers in this group were very traditional in nature. This teacher spent a great deal of time with other teachers in the school's mathematics department to find additional resources to help them sequence learning and provide their students with more complex problems and activities.

The approved textbook for the other teacher in this group was not traditional. It was activity based, filled with explorations, and provided problems that require more than rote application of mathematics algorithms. As the only teacher in the department, this teacher relied heavily on the textbook, but found ways to support the text with sample problems for students who need more practice with skills. She credits the textbook with contributing to the development of her philosophy of mathematics teaching and learning. She explained, "It's not how I learned, certainly. I definitely learned from skill and drill and watching and repeating, so I definitely learned the benefits of [active learning] through the years... I think it

became my philosophy." Still, she acknowledged the need to supplement this textbook with skills-based problems.

When asked about their school's philosophy of teaching and learning, each of the teachers in this group described their school's emphasis on preparation for life after high school, whatever path the student might choose. In both schools, a successful student is not necessarily a student who has demonstrated mathematics achievement on tests but, rather, one who is well rounded and has developed important life skills, such as reading, writing, and critical thinking. One of the teachers indicated that many of the students do not go to college, and her school provides opportunities for students interested in pursuing a variety of pathways.

Student-Centered Approach to Instruction

One teacher in our case study sample implemented student-centered approaches almost exclusively. Throughout the development phase of the lesson, students typically worked in small groups on exploratory activities that were carefully designed to support discovery of the new mathematical rule, procedure, or relationship. After the exploration, students would come together as a whole group to share their findings and work as a whole class to finalize the new mathematical concept. Problems used during the reinforcement and extension phase required critical thought and mathematical reasoning. Throughout both phases, the teacher used open-ended questions to encourage students to communicate and justify their thinking, critique others' thinking, and make connections.

When asked about her philosophy of mathematics teaching and learning, she explained:

"I like it to be relevant. I like kids to see the application of what they're doing. Whether that's a real-world application or if it's just something really cool within mathematics,

you know, check out this relationship, you know that kind of thing. It can still be pure math, but if it's something that's relevant or elegant or whatever, it's worth studying. I don't like to drill and kill. I think it's important for them to have conversations with each other around math just to process information. I don't like it when the room is totally silent when they're supposed to be working on math, unless it's a time when I give them an individual assessment. Then they need to be silent, but if they're supposed to be working together and it's silent, I worry about what they're doing."

As they work in groups, this teacher likes for her students use two-foot by two-foot whiteboards. Work on the whiteboards facilitates the kind of activities that she believes are important to student learning. She explained:

"The whiteboards are nice because it gives them a common piece of paper... and I try to get them to lay it flat so everybody can see it, so there isn't one person hogging the whiteboard. It also gives them something, a focal point, and it allows me to see fairly easily, not just how the group is working, but what their work is, what their progress is. Depending on what the problem is or the topic or the purpose for the work, sometimes I will just let them do what they do, right or wrong, and then... all five whiteboards will go up on the front tray, and we'll take a look at them. We kind of debrief the work, and the first question is: What do you notice? Do we all agree?"

We observed one class period in which the lesson summary consisted of students examining each other's whiteboards, arguing which approaches were consistent and correct, and which were not and why. The students were able to reach resolution on their own, with very little input from the teacher. The teacher was an active observer in this summary discussion, but the energy behind the lively debate came from the students.

To plan her lessons, this teacher relies on the textbook, which is highly activity based. The text contains numerous explorations and problems that require mathematical reasoning, communication of mathematical thinking, and opportunities to critique others' reasoning. She spends time with other teachers in the department looking closely at the activities in the text and discussing what has and hasn't worked with students in the past. From there, she makes decisions regarding which activities to use with her students.

When asked about the school's philosophy of teaching and learning, she replied, "Teach kids to use their minds well—teach them how to learn, how to work with others, teach them how to be good people. Everybody has skills. Everybody has strengths. We're here to find out what your strengths are and work on your weaknesses." She indicated that, while students at the school do not always score well on standardized tests and/or go to college, they are kind to each other. They care about each other. In that respect, they are considered very successful.

Themes Across Teachers with Varying Degrees of Student-Centered Instruction

In comparing teachers who use student-centered approaches to varying degrees, we see some differences both in philosophy and instructional environment. In contrast to teachers who more regularly use traditional approaches, those who more regularly use student-centered approaches cited the importance of instructional approaches in which students explore mathematics, communicate their thinking, and have the opportunity to reason and critique others' reasoning, with some opportunity to practice skills. In contrast to more traditional teachers, these teachers work in schools that, from their perspective, focus on preparing students for a variety of pathways, not always connected to attending college. From their perspective, success in their schools is not measured by test scores but by a variety of life skills.

“*Teacher beliefs and flexibility in lesson design may contribute to how regularly student-centered approaches are implemented.*”

Although we have a small sample of teachers, we did notice some patterns between the type of textbook used and instructional style. Three of the teachers who more regularly implemented traditional approaches used a traditional textbook, and the teacher who employed student-centered approaches almost exclusively used a textbook full of exploratory activities and complex problems. The two teachers who implemented a blend of traditional and student-centered approaches used both types of textbooks. One of these teachers used a textbook that was similar to the teacher who implemented student-centered approaches more regularly, while the other used traditional textbooks with student-centered activity supplements from other sources. These cases illustrate that, although certain

kinds of textbooks can better support student-centered instruction, there are other ways for teachers to integrate more student-centered approaches, even if the school has not adopted such a textbook.

Our study suggests that teachers who: (1) believe in the importance of instructional approaches that provide opportunities for students to engage meaningfully with mathematics; (2) work in schools that focus on preparing students for a variety of life pathways; and (3) have flexibility in lesson design may implement student-centered approaches more regularly than teachers who do not have all three of these things in place. ■■■

FINDINGS PART THREE

STUDENT PERSPECTIVES

Our third research question addressed students' perceptions of these different types of instructional approaches. In particular, we were interested in learning what students felt had helped them be successful and whether or not their opinion of mathematics had changed as a result of being in the class. To collect this information, we interviewed a subset of students identified by each case study teacher as having shown growth in engagement or learning over the course of the year.

When asked what helped them to be successful, students of the three teachers who used more traditional approaches said that they appreciated the highly structured, well-organized nature of their classrooms. They indicated that they liked that they would take notes before they did practice problems. They also cited their teachers' ability to explain things clearly and in multiple ways and willingness to help students when they needed it. These sentiments are evidenced in the following statements:

“He always prepared PowerPoints for us so like all of our notes are organized and everything.”

“Taking notes before we have homework, that really helps with everything.”

“If you need to do a problem, she’ll do it on the board, and if you still don’t get it, then she’ll try to explain it in different ways... She tries to understand how you know things, and what you don’t know, and she tries to explain it to you.”

“The way she says it makes sense in your head.”

When asked what helped them to be successful, the students in classes with the three teachers who used student-centered approaches more regularly commented on the use of more interesting activities:

“It’s not the same routine every day.”

“She connects things to real life.”

“We’ll build stuff, and that’s the only way I can get it.”

“She always has everybody engaged, not just certain people answering questions.”

When asked whether (and how) their opinions of mathematics changed as a result of being in this class, both groups commented that their opinions had changed.

Students in more traditional classrooms indicated that they felt more confident in their mathematical abilities and felt that mathematics was now “more manageable.” Although these students felt better about mathematics, they didn’t go so far as to say they actually enjoyed the subject. On the other hand, students in more student-centered classrooms said that they no longer dreaded mathematics and some even enjoyed it:

“She made me enjoy math more than I usually do... Usually I dreaded math class because it was so boring but, this one’s hands on, there’re different things that are going on. It’s more active than any other math class I’ve been in.”

“With all the group work and all that, now I understand it. I don’t hate it as much as when I started.”

Overall, regardless of instructional approach, the students felt very positively about their experience in the teachers’ classrooms, which is perhaps not surprising since our sample consisted entirely of highly-regarded teachers. These types of teachers are organized and go out of their way to help students feel better about their abilities and be successful. Their students recognize and appreciate these attributes and practices. However, our data suggest that students assigned to teachers with more student-centered approaches also appreciate specific things about their classrooms. They reported finding the content interesting and meaningful, and some had grown to love mathematics.



RELATIONSHIP TO STUDENT OUTCOMES

When we designed the study, we wanted to not only understand student-centered high school mathematics teaching more deeply, but also to see whether there was a difference in student engagement and achievement in classrooms with varying degrees of student-centered instruction. To answer our final research question, we developed a problem-solving assessment and student survey, which were administered to students in both case study and non-case study classrooms at the end of the 2012–13 school year. We used quantitative methods to analyze the relationship between the instructional approach and the survey and assessment data. Before we present the results of these analyses, we provide more details about each independent and dependent variables, how they were measured, and how they were used in the analyses.

Composite Measure of Student-Centered Mathematics Instruction

To provide an indicator of the degree to which teachers implemented (and students experienced) student-centered practices (SCP) to mathematics instruction, we used the data from the teacher survey and the challenging assignments submitted by all 22 teachers. Because the teacher survey and assignment data applied to each teacher’s target class, these data reflect the learning environments of the same students who took the problem-solving assessment and completed the survey, making this indicator appropriate to include as an independent variable in our quantitative models.

For the teacher survey, we focused on questions that provided information on the extent to which each teacher

provided opportunities for students to meaningfully engage with mathematics. For example, teachers reported how often they used “exploratory activities” and how often “students were expected to analyze and respond to other students’ thinking.” For these and other relevant survey items, the frequency with which teachers reported providing these opportunities ranged from “never” to “every day or almost every day.” From these response options, we created a numeric indicator, SCP_{TS}. Responses were assigned a number from 0 to 3 (with “never” receiving a 0 and “every day or almost every day” receiving a 3). The numbers were then added up, and a percentage of total points received out of the total possible represented the SCP_{TS}.

The teacher survey collected information about the extent to which teachers provided three of the four opportunities for meaningful engagement with mathematics described throughout this report: using mathematical reasoning to understand the “why” as well as the “how”; communicating mathematical thinking and critiquing the reasoning of others; and making connections between and among mathematical concepts and real-world contexts. To gain insight into the fourth opportunity to meaningfully engage with mathematics, the extent to which students had opportunities to engage and persevere in solving non-routine mathematical problems, we analyzed teachers’ challenging assignments.

At four points in the year, teachers were asked to submit the most challenging assignment that students in their target class had been asked to complete during the prior time period. We scored these assignments on the extent to which students were expected to demonstrate conceptual understanding, critical thinking, and effective communication skills, as well as the degree to which

the assignment emphasized connections between mathematical concepts and real-world contexts. Each assignment received a score that represented points earned out of the total possible (15 points). We then created an indicator of SCP for the challenging assignment (SCP_{CA}) by averaging the four assignment scores.

We combined the SCP_{TS} with the SCP_{CA} to create an overall SCP for each teacher. We applied a weight of 75% to SCP_{TS} and 25% to SCP_{CA} , since the survey represented 3 of the 4 opportunities for meaningful engagement and the challenging assignments for 1 of these 4 opportunities.

Student Engagement

To construct a measure for student engagement, we drew items from the survey that was administered to students in all 22 study classrooms. The survey included two constructs related to engagement. The first measured student's self-assessment of learning and included two survey items: "This math class really makes me think" and, "I'm learning a lot in this math class." The second construct measured

student interest and motivation to participate and was comprised of three items: "I usually look forward to this math class," "I work hard to do my best in this math class," and "In this math class, sometimes I get so interested in my work I don't want to stop." We combined students' ratings on each response into a joint score for each construct.

We then compared these constructs to the following mixed model:

$$Y_{SS} = a + SCP \cdot \delta + \epsilon$$

In this equation, Y_{SS} is a vector of the students' score on that construct (engagement or interest), SCP is the composite measure of student-centered instruction described previously, a is a regression coefficient for the intercept, δ is a regression coefficient for the effect of student-centered teaching on the students' engagement or interest, and ϵ is an error term that includes a component for the teacher using a linear mixed model. This model measures survey outcomes as a function of the SCP measure and does not adjust for any baseline covariates.

Problem-Solving Assessment

Our problem-solving measure consisted of nine published items from the 2009 PISA. As illustrated in Figure 7, these items require more than strong computation and procedural skills. They require students to interpret a problem situation, apply their knowledge of specific mathematical concepts and skill to that situation, and explain how they arrived at

the answer. PISA focuses on problem solving, rather than a particular curriculum or course, making the assessment appropriate for a study that includes students enrolled in a variety of mathematics courses. The characteristics of this problem-solving assessment were also well aligned with the study's definition of meaningful engagement with mathematics.

Figure 7. Examples of the Study's Mathematics Problem-Solving Assessment

EXCHANGE RATE

Mei-Ling from Singapore was preparing to go to South Africa for three months as an exchange student. She needed to change some Singapore dollars (SGD) into South African rand (ZAR).

Mei-Ling found out that the exchange rate between Singapore dollars and South African rand was 1 SGD = 4.2 ZAR. Mei-Ling changed 3,000 Singapore dollars into South African rand at this exchange rate.

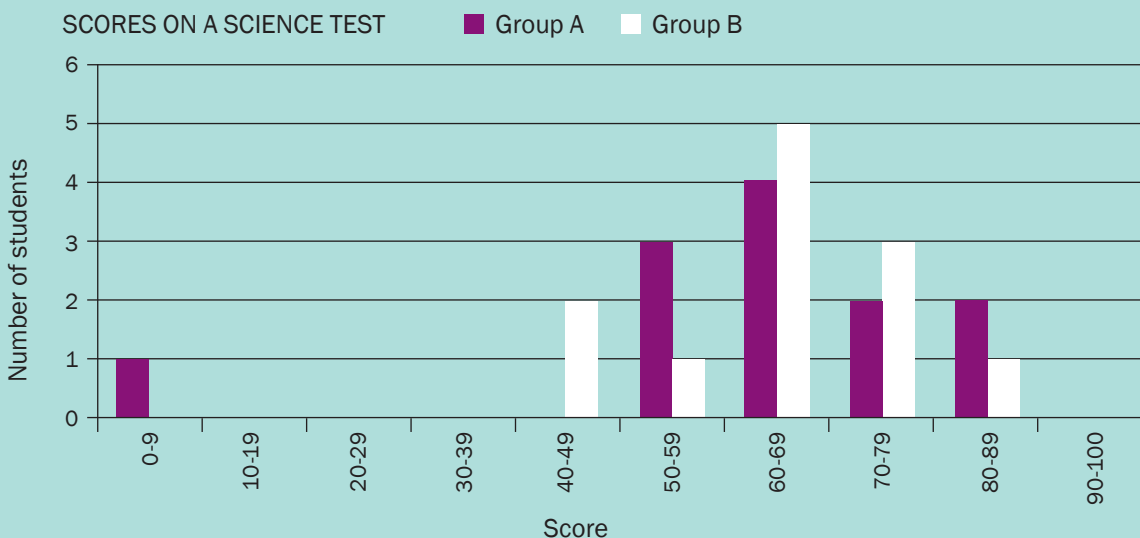
HOW MUCH MONEY IN SOUTH AFRICAN RAND DID MEI-LING GET?

On returning to Singapore after three months, Mei-Ling had 3,900 ZAR left. She changed this back to Singapore dollars, noting that the exchange rate had changed to 1 SGD = 4.0 ZAR. How much money in Singapore dollars did Mei-Ling get?

During these three months, the exchange rate had changed from 4.2 to 4.0 ZAR per SGD. Was it in Mei-Ling's favor that the exchange rate now was 4.0 ZAR instead of 4.2 ZAR, when she changed her South African rand back to Singapore dollars? Give an explanation to support your answer.

TEST SCORES

The diagram below shows the results on a science test for two groups, labeled as Group A and Group B. The mean score for Group A is 62.0, and the mean for Group B is 64.5. Students pass this test when their score is 50 or above.



Looking at the diagram, the teacher claims that Group B did better than Group A in this test. The students in Group A don't agree with their teacher. They try to convince the teacher that Group B may not necessarily have done better. Give one mathematical argument, using the graph that the students in Group A could use.

Similar to the procedures that PISA uses, we assigned each item a difficulty level, and students were given scaled scores, using the Rasch model. This model was appropriate for the study's nonrandom sample of teachers and students because Rasch scores can be accurately generated on non-random samples. Because only a small number of items were used to keep the time requirements reasonable, the reliability of the problem-solving assessment was lower than typical of a state assessment, which is usually near 0.90. The study's problem-solving assessment still has a reasonably high reliability of 0.76, which proved sufficient for our study purposes.⁶

Before investigating the differences between exemplary classrooms, it is worth noting that students in the study, on average, performed better than U.S. students on all nine PISA items. For example, in the exchange rate problem (Figure 10), about 80 percent of the study students answered each of the first two questions correctly, compared with 54 percent and 68 percent of U.S. students, for each respective question. For the test scores problem, 53 percent of the study students answered this question correctly, compared with 40 percent of U.S. students. We also compared the performance of students in the study with the overall international average, which was higher than the U.S. average for all but one of the nine items on the assessment. On all but one item, the study students performed better than the international average, and they scored only one percentage point lower than the international average on the remaining item. The study was not designed to determine why students in the study performed better on average than the U.S. and international averages, but the results do show that students in exemplary

classrooms did relatively well on these items.

Outcome scores cannot be directly compared because they may reflect a difference in baseline achievement. To account for this, we constructed a value-added model that incorporated students' prior math achievement as measured by their eighth-grade mathematics test score and used the PISA scaled score as the outcome. The eighth-grade achievement measures included: the Connecticut Mastery Test (two teachers), Massachusetts Comprehensive Assessment System (one teacher), New England Common Assessment Program (16 teachers), New York State Testing Program (one teacher), and Secondary School Admission Test (one teacher). Because the only eighth-grade achievement measures associated with more than two teachers was the New England Common Assessment Program (NECAP), we estimated our value-added model using only those 16 teachers and their students. The model, then, took the form:

$$Y_{PISA} = a + Y_{NECAP-8} \cdot \beta + SCP \cdot \delta + \epsilon$$

In this equation, Y_{PISA} is a vector of the students' end of year PISA scale scores, $Y_{NECAP-8}$ is the vector of students' grade eight math NECAP scale scores, SCP is the composite student-centered instruction measure defined previously, a is a regression coefficient for the intercept, β is a regression coefficient for the slope of the grade eight NECAP, δ is a regression coefficient for the effect of student-centered teaching on the students, and ϵ is an error term that includes a component for the teacher using a linear mixed model. We used this model to compare the value-added results of students in classrooms with varying degrees of student-centered instruction, using the study's SCP measure. ■■■

6 State assessments have reliabilities closer to 0.90, but they usually take substantially longer to administer than the study's 45-minute assessment

RESULTS

Using our SCP indicator of student-centered mathematics instruction, the main results related to the study's final research question are as follows:

- Students in more student-centered classrooms reported higher levels of engagement and interest on their surveys.
- Students in more student-centered classrooms also had higher PISA assessment scores in a value-added framework, which controls for prior mathematics achievement.

Focusing on the student survey, we found statistically significant positive relationships (p-values of approximately 0.002) between the SCP measure and both survey constructs for student engagement: student self-assessment of learning⁷ and student interest.⁸

For the problem-solving assessment, we found that students in classrooms taught by teachers with higher SCP scores showed more growth on the PISA than students in classrooms taught by teachers with lower SCP scores. Here an increase of 0.01 on the SCP scale (a 1% increase) is associated with an increase of 0.02 scaled score on the PISA. While it is difficult to interpret this coefficient,⁹ the increased rate of growth for students in classrooms taught by teachers with higher SCP scores is statistically significant, with an associated p-value just under 0.05.¹⁰ Taken together, these results indicate that the benefits to students of having a highly regarded mathematics teacher are even greater when the teacher implements student-centered approaches to instruction as defined in this study.

CONCLUSION & IMPLICATIONS

Several conclusions emerged from our mixed-methods study of student-centered mathematics instruction. Our qualitative analyses identified a range of instructional approaches that provided opportunities for students to engage meaningfully with mathematics. Through our data collection activities, both as part of recruitment and as part of the study, we gained an even stronger appreciation than we previously had for the complexity of mathematics instruction.

Our data provides evidence that it is not possible to classify mathematics instruction in only two categories: traditional and student-centered. There are many variations in the types and frequencies with which teachers implement student-centered approaches in mathematics classrooms. Our study showed that even teachers who report implementing more traditional approaches to instruction will often implement some aspects of student-centered instruction at times. We also learned that the instructional context—the philosophy of the teachers and the school, the curricular materials available—may be related to the degree to which teachers use more traditional or student-centered approaches, but the relationships are not always straightforward. For example, we found student-centered approaches being implemented with more traditional curricular materials and traditional approaches being implemented with more activity-based programs. We also learned that students appreciate being taught by highly regarded teachers and are able to identify specific aspects of instruction that help them succeed in both traditional and student-centered classrooms.

7 As a reminder, this construct sums survey responses for includes two items: "This math class really makes me think" and, "I'm learning a lot in this math class."

8 Again, repeating from above, this construct sums up responses to three items: "I usually look forward to this math class," "I work hard to do my best in this math class," and "In this math class, sometimes I get so interested in my work I don't want to stop."

9 It is difficult to interpret because the typical approach of comparing the change to a Z-score change is not possible. For this study, we do not have a representative sample of students or teachers to accurately create a population-based Z-score of either SCP or the PISA scale scores.

10 Both of the p-values for the student survey and PISA assessment suggest that neither positive relationship is likely the result of random statistical fluctuation.

Our quantitative analyses showed positive, significant relationships between the study's measure of student-centered practices and students' engagement and problem-solving skills, suggesting that the benefits of having a highly regarded mathematics teacher may be even greater if the teacher is more student-centered in his or her approach.

Drawing on these conclusions, we think this study has at least three concrete implications for policymakers and practitioners who are interested in promoting student-centered instruction in mathematics.

A more fine-grained definition of student-centered mathematics instruction may help promote this type of instruction.

This study demonstrates that a construct as multi-faceted and complex as student-centered instruction can be more fully understood when certain aspects of teaching are isolated. This observation may be important for practitioners who are trying to understand what it takes to teach mathematics for understanding, a goal which will grow in importance as states move closer to implementing more rigorous, high-stakes assessments associated with the Common Core State Standards for Mathematics (CCSM). For teachers who are new to principles of student-centered instruction and view the endeavor as daunting, our study suggests that there are multiple entry points to this approach, which also means there are potentially multiple entry points for providing support. For example, a teacher might decide to implement an activity-based task and focus on supporting students' conceptual understanding as a first step, rather than taking on the art of open-ended questioning. Narrowing student-centered instruction to the types of activities and discussions we've outlined for the development and reinforcement stages of instruction could help in establishing and monitoring goals for improvement.

There appears to be an interaction between mathematical content and modes of student-centered instruction.

This study focused exclusively on high school mathematics, which includes theoretical topics that may not naturally lend themselves to some student-centered approaches. If the study had focused on K-8 mathematics or elementary science, for example, the opportunities for student-centered instruction may have been greater. Despite the inherent challenges of making certain high school mathematics topics and concepts seem engaging and relevant, our study showed that it is possible to do this at the high school level. This study showed that teachers can still use student-centered approaches when opportunities for making real-world connections do not exist or are, at best, a stretch by varying their approaches depending on the topic. For instance, some of the study's teachers were able to maximize students' understanding of highly abstract concepts by building upon students' prior knowledge when topics did not lend themselves to applied activities. The variation in approaches we observed suggests that teachers may need to draw from a variety of a methods, considering which student-centered approaches will be most suitable to a given topic as they plan and deliver lessons.

Teaching philosophy and instructional context may interact and affect how strongly and consistently teachers enact student-centered approaches.

While a more fine-grained definition of student-centered instruction has the potential to provide more entry points for teachers to attempt such a shift, a teacher's philosophy of mathematics teaching and instructional context must also be carefully considered. Our study suggests that teachers who believe that activity-based instruction is important, work in schools that prepare students for different types

of pathways, and have flexibility in lesson design tend to implement student-centered approaches more strongly and consistently. Instructional coaches and administrators should consider a teacher's philosophy and instructional context before setting targets for implementing student-centered approaches. Drawing on such information could help their efforts be even more targeted and impactful. There is still much more we can learn. This study demonstrated that it is possible to learn a lot about

student-centered mathematics instruction with a small sample of highly regarded teachers, but this is only one study design. The field of mathematics education could benefit from future studies that focus on different grade levels, topical areas, school systems, and regions of the country. With more contributions like those of the dedicated, highly regarded teachers who graciously participated in this study, the prospects for what the field can still learn are bright. ■■■

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